Faculty of Engineering Chulalongkorn University

2110355 Formal Languages & Automata Theory

First Semester of 2002-03

1st exercise for Wednesday 12th June 2002 (12:45-14:00)

1. Let S_a, S_e, S_i, S_o, and S_u be the fives set of words in English Language where

   $$S_x = \{y : y \text{ is a word containing } x\}$$

   given that x is either a, e, o, or u.
   Write down the sets of following English words in the form of the 5 sets
   (i) Words which contain either 'a', 'e' or 'i'.
   (ii) Words which contain at least an 'a', an 'o', and a 'u'.
   (iii) Words which contain an 'e', an 'o' and a 'u'.
   (iv) Words which contain neither 'e' nor 'i'
   (v) Words which contain 'a' and 'e' but not 'i'.

Closure: Let A be a set. The closure of A, denoted by A* is the set of all sequences of elements of A including empty element Λ. For instance, A = {0, 1}, we have that A* = {Λ, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, …}.

2. Let L_1 and L_2 be languages over *{0, 1}* ($L_1 \subseteq \{0, 1\}^*$ and $L_2 \subseteq \{0, 1\}^*$). **Prove or disprove that if $L_1 \subseteq L_2$, then $L_1^* \subseteq L_2^*$.**

3. For a function $f : A \rightarrow B$ and $A_1 \subset A, A_2 \subset A$
   (i) Prove that
   $$f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$$
   (ii) Prove that
   $$f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$$
   (iii) Give and example when equality holds in (ii)

4. Let R be the relation defined on set $\Re$ (real number). R is defined as:
   for $x, y \in \Re$, $x\ R\ y$ if x-y is an integer. Show that R is an equivalence relation.

5. Let R be the relation defined on the set of **all** languages in *{0, 1}**. R is defined by:
   $L_1\ R\ L_2$ if and only if there exists a string x such that $x \in L_1 - L_2$.
   Proof or disproof that *R* has the following properties:
   (i) reflexive, (ii) symmetric, (iii) transitive.

6. Let a relation R defined on Σ^* where $\Sigma = \{a, b\}$. For $x, y \in \Sigma^*$:
   $x\ R\ y$ if and only if $|x| = |y|$. Prove that R is an equivalence relation

7. Let $L_1 = \{ x \in \{0, 1\}^* \mid n_a(x) = n_b(x)\}$.
   x and y be strings in L_1. And R is defined by:
   x R y if and only if $n_a(x) \leq n_b(y)$.
   Proof or disproof that *R* has the following properties:
   (i) reflexive, (ii) symmetric, (iii) transitive.

8. Let C be the set of provinces in Thailand
   Define: *a R b* to hold for two provinces in C where there is a non-stop bus trip from a to b

   (i) Is R reflexive?
   (ii) Is R transitive? Justify your answer

9. Prove that if $n+1$ distinct integers are selected from the set $\{i : 1 \leq i \leq 2n\}$ the one of the selected integers will divide some other selected integer exactly.

10. To check whether an integer n, where $n > 1$, is prime, it suffices to check that n is not divisible by any prime number less than or equal to $\sqrt{n}$. Justify the statement.

11. For all positive integer n, $n^2 - 85n + 249$ is prime. Prove this statement or give a counterexample to disprove it.

12. For each each $n \geq 0$, we define the strings a_n and b_n as follows:
    $a_0 = 0$ $b_0 = 1$ for $n > 0$, $a_n = a_{n-1}b_{n-1}$; $b_n = b_{n-1}a_{n-1}$

    (i) For every $n \geq 0$, the string a_n and b_n are of the same length.
    (ii) For every $n \geq 0$, a_n and b_n differ in every position.
    (iii) For every $n \geq 0$, a_{2n} and b_{2n} are palindromes.
    (iv) For every $n \geq 0$, a_n contains neither the substring 000 nor the substring 111.

13. Show that the sum of any three consecutive integers is divisible by 3. (Two integers are consecutive *iff* one integer differs from the other by the value of one)

14. Any two consecutive integers have opposite parity.

15. Use mathematical induction to prove that for all positive integer n
    $2 + 4 + 6 + \ldots + 2n = n^2 + n$ is true

16. Show that for all integers $n \geq 1$, $2^{2n} - 1$ is divisible by 3.

17. Observe that,

$$\frac{1}{1\times 3}=\frac{1}{3}$$
$$\frac{1}{1\times 3}+\frac{1}{3\times 5}=\frac{2}{5}$$
$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}=\frac{3}{7}$$
$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\frac{1}{7\times 9}=\frac{4}{9}$$

Guess a general formula and prove it by mathematical induction.

18. For each n ≥ 0, we define the strings a_n and b_n as follows
$a_0 = 0$ $b_0 = 1$ for $n > 0$, $a_n = a_{n-1}b_{n-1}$; $b_n = b_{n-1}a_{n-1}$

(i) For every n ≥ 0, the string a_n and b_n are of the same length.
(ii) For every n ≥ 0, a_n and b_n differ in every position.
(iii) For every n ≥ 0, a_{2n} and b_{2n} are palindromes.
(iv) For every n ≥ 0, a_n contains neither the substring 000 nor the substring 111.