6

จำนวนสถานะ

NUMBER OF STATES

NUMBER OF STATES

ประเด็นที่สนใจ

The number of distinct states the finite state machine needs in order to recognize a language is related to the number of distinct strings that must be distinguished from each other.

NUMBER OF STATES

สามารถแยกความแตกต่างได้

DISTINGUISHABLE

นิยาม

Let L be a language in Σ^*.
Two strings x and y in Σ^* are distinguishable with respect to L if there is a string z in Σ^* so that exactly one of the strings xz and yz is in L.

The string z is said to distinguish x and y with respect to L.

NUMBER OF STATES

สามารถแยกความแตกต่างได้

DISTINGUISHABLE

นิยาม

Let L be a language in Σ^*.
Two strings x and y in Σ^* are
distinguishable with respect to L
if $L/x \neq L/y$ where

$$L/x = \{ z \in \Sigma^* \mid xz \in L \}$$
$$L/y = \{ z \in \Sigma^* \mid yz \in L \}.$$

NUMBER OF STATES

สามารถแยกความแตกต่างได้

DISTINGUISHABLE

EXAMPLE

Let Σ = { 0, 1 }.
Let L be the language associated with (0+1)*10.

Two strings x= 01101 and y= 010 in Σ*.
Since there is a string z= 0 in Σ* such that
xz = 011010 is in L but yz = 0100 is not in L,
x and y are distinguishable with respect to L.

We may say that x and y are indistinguishable with respect to L if there is no such string z.
The strings 0 and 100 are indistinguishable with respect to L.

NUMBER OF STATES

จำนวนสถานะ

NUMBER OF STATES

LEMMA

Suppose that $L \subseteq \Sigma^*$, and $M = (Q,\Sigma,q_0,A,\delta)$.
If x and y are two strings in Σ* for which

$$\delta^*(q_0,x) = \delta^*(q_0,y)$$

then x and y are

indistinguishable with respective to L.

Note: $\delta^*(q_0,x)= q_j$ means that there is a path from q_0 to q_j with respect to x:

$$\delta^*(q_0,x) = \delta((\ldots\delta(\delta(q_0,x_1),x_2),\ldots),x_j) = q_j$$

where $x = x_1x_2\ldots x_j$.

NUMBER OF STATES

จำนวนสถานะ

NUMBER OF STATES

LEMMA

Suppose that $L \subseteq \Sigma^*$, and $M = (Q,\Sigma,q_0,A,\delta)$.
If x and y are two strings in Σ^* for which

$$\delta^*(q_0,x) = \delta^*(q_0,y)$$

then x and y are

indistinguishable with respective to L.

Proof: Let z be any string in Σ^*. Consider xz and yz,
We have that $\delta^*(q_0,xz) = \delta^*(\delta^*(q_0,x),z)$
$\delta^*(q_0,yz) = \delta^*(\delta^*(q_0,y),z)$.
Then $\delta^*(q_0,xz) = \delta^*(q_0,yz)$. Two strings xz and yz are either both in L or both not in L.
Therefore, x and y are indistinguishable with respect to L. QED.

NUMBER OF STATES

จำนวนสถานะ

NUMBER OF STATES

THEOREM

Suppose that $L \subseteq \Sigma^*$, and for some positive integer n, there are n strings in Σ^*, any two of which are distinguishable with respect to L.

Then there can be no finite state machine recognizing L with fewer than n states.

Proof: Suppose $x_1,x_2,\ldots,x_n$ strings are distinguishable with respect to L. Assume that M is a finite state machine with fewer than n states. By the pigeonhole principle, the state $\delta^*(q_0,x_1)$, $\delta^*(q_0,x_2),\ldots\delta^*(q_0,x_n)$ cannot all be distinct, so for some $i \neq j$ $\delta^*(q_0,x_i)=\delta^*(q_0,x_j)$.
Since x_i and x_j are distinguishable with respect to L, it follows from Lemma that M cannot recognize L. QED.

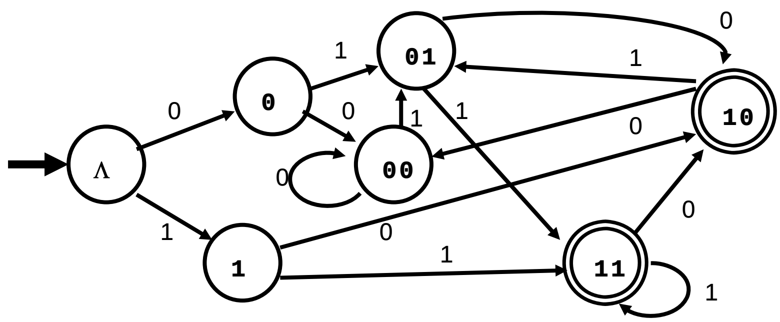
NUMBER OF STATES

จำนวนสถานะ

NUMBER OF STATES

EXAMPLE

(1+0)*1(0+1)



NUMBER OF STATES

จำนวนสถานะ

NUMBER OF STATES

THEOREM

> PALINDROME language over the alphabet {0,1} cannot be accepted by any finite automaton.

Proof:

Any two strings in {0,1}* are distinguishable with respect to PALINDROME language. QED.

EXAMPLE

For two strings, 010101 and 1011100 — 11101 — Rejected / accepted

For two strings,	010101	11101	Rejected
and	1011100		accepted