

9

KLEENE'S  
THEOREM

KLEENE's THEOREM

วัตถุประสงค

**UNIFICATION**

Kleene's theorem, 1956

All three methods of defining languages  
regular expression,  
acceptance by finite automata and  
acceptance by transition graph  
are equivalent

**KLEENE's THEOREM**

# วัตถุประสงค

**It is clear that a finite automaton is a transition graph.**

## **UNIFICATION**

Kleene's theorem, 1956

- Language accepted by a finite automaton can be defined by a transition graph.
- Language accepted by a transition graph can be defined by a regular expression.
- Language generated by a regular expression can be defined by a finite automaton.

**KLEENE's THEOREM**

## **UNIFICATION**

Kleene's theorem, 1956

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Proof:

Given a transition graph TG.

Find a regular expression that defines the same language.

Construct an algorithm that satisfies two criteria.

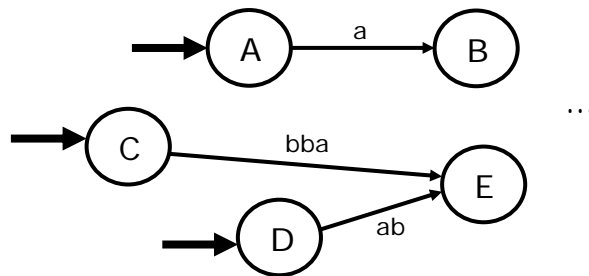
- Work for every conceivable transition graph
- Guarantee to finish its job in a finite time.

KLEENE's THEOREM

# UNIFICATION

Kleene's theorem, 1956

**STRATEGY**  
Only one initial state.

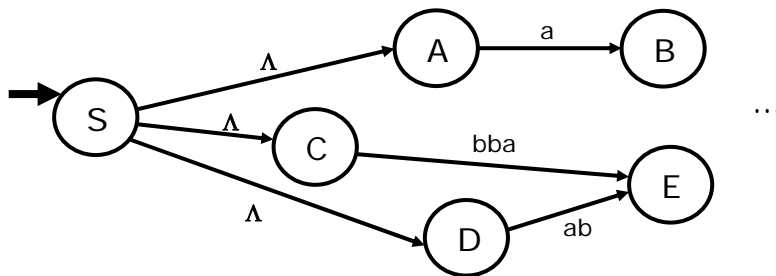


KLEENE's THEOREM

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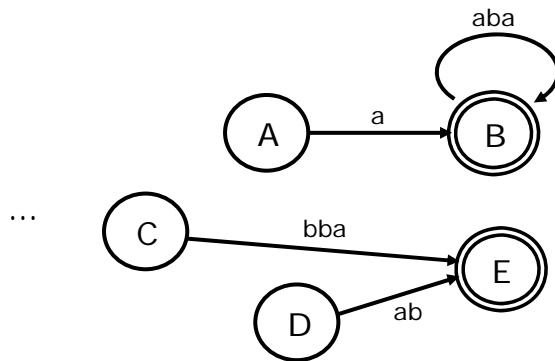


KLEENE's THEOREM

# UNIFICATION

Kleene's theorem, 1956

**STRATEGY**  
Only one final state.

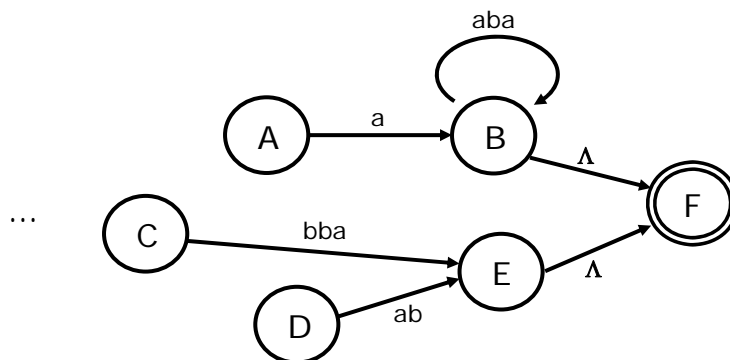


KLEENE's THEOREM

# UNIFICATION

Kleene's theorem, 1956

**STRATEGY**  
Only one final state.





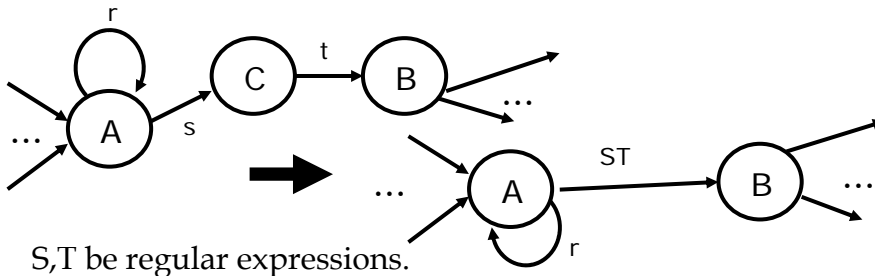
KLEENE's THEOREM

# UNIFICATION

Kleene's theorem, 1956

**STRATEGY**  
Eliminate all  
internal states.

Let  $A, B$  be states in a transition graph.  
 $r, s, t$  and  $u$  be any substrings.



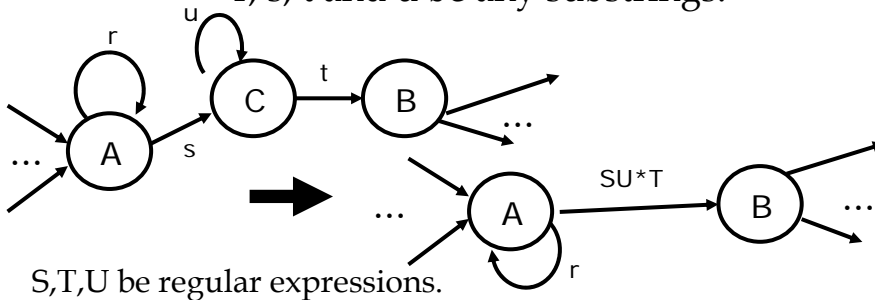
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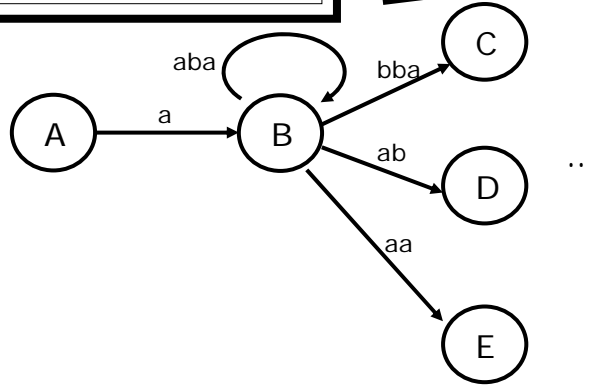


KLEENE's THEOREM

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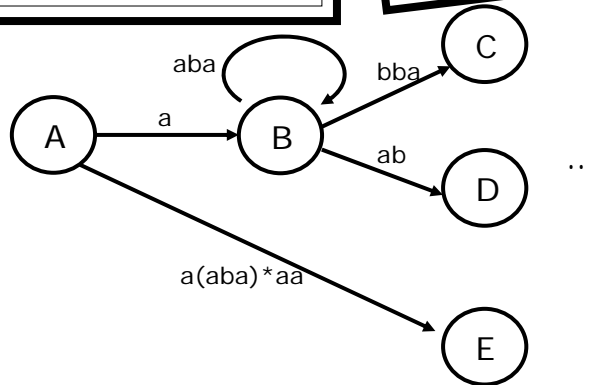


KLEENE's THEOREM

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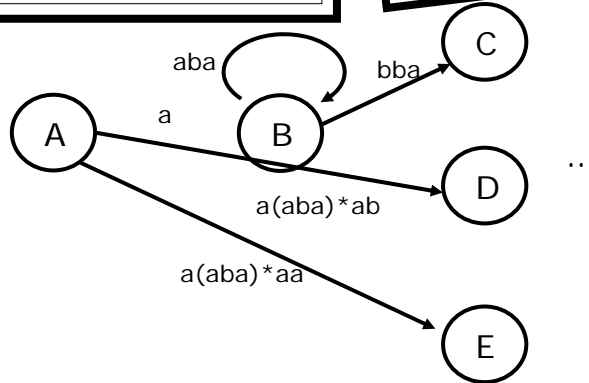


KLEENE's THEOREM

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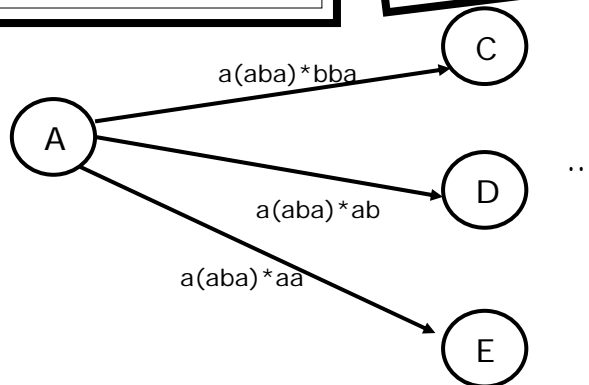


KLEENE's THEOREM

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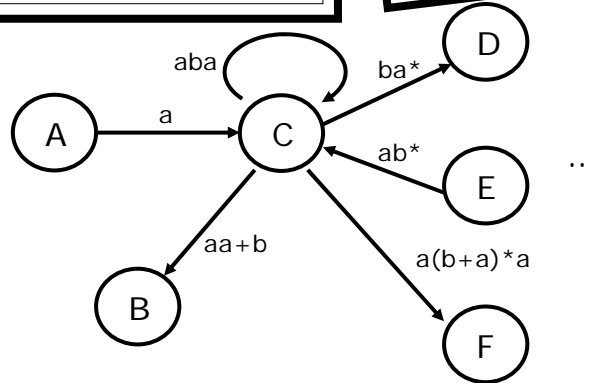


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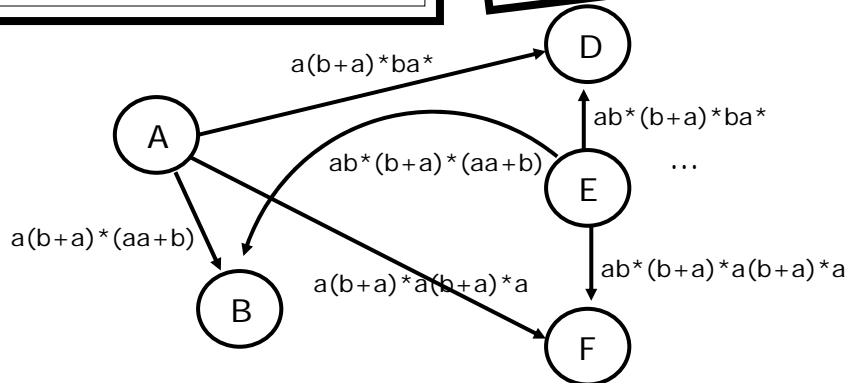


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Kleene's theorem, 1956

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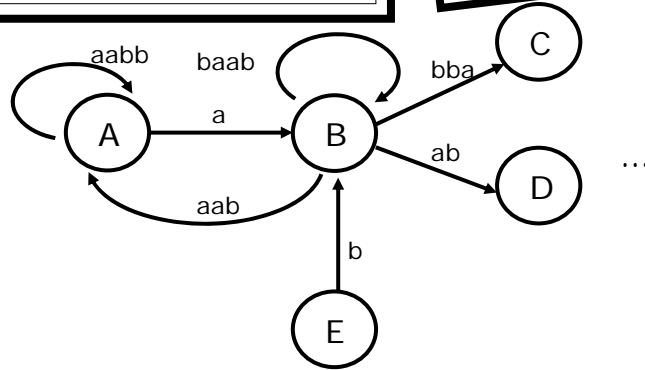


**KLEENE's THEOREM**

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Kleene's theorem, 1956

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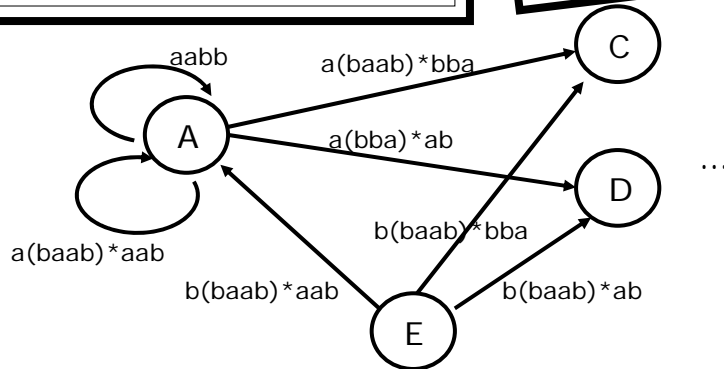


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Kleene's theorem, 1956

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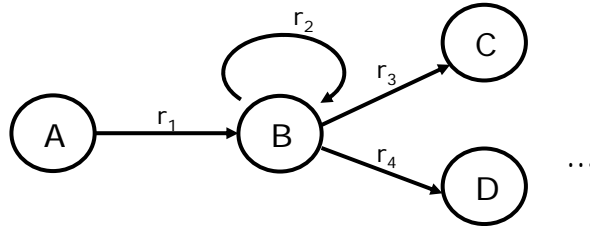


KLEENE's THEOREM

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Kleene's theorem, 1956

**STRATEGY**  
Eliminate all  
internal states.



Eliminate B, by

create  $AC = r_1 r_2^* r_3$

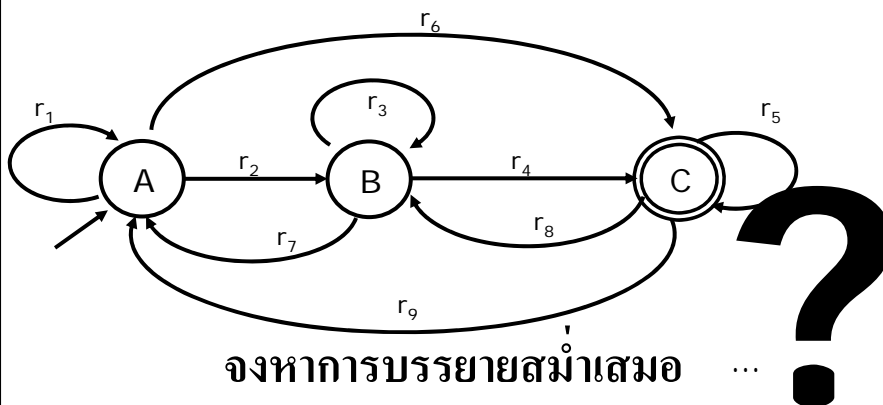
create  $AD = r_1 r_2^* r_4$

delete AB

erase B when B has no input.

โจทย์

น้ำคิด



จงหาการบรรยายสม่ำเสมอ

regular expression

KLEENE's THEOREM

# UNIFICATION

Kleene's theorem, 1956

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Proof:

Given a regular expression.

Find an automaton that defines the same language.

Construct an algorithm that satisfies two criteria.

- Accepts any particular letter of the alphabet. (or  $\Lambda$ )
- Close under  $+$ , concatenation and Kleene's star.

KLEENE's THEOREM

# UNIFICATION

Kleene's theorem, 1956

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Let  $FA_1$  accepts the language defined by regular expression  $r_1$ ,

$FA_2$  accepts the language defined by regular expression  $r_2$ .

Then there is a  $FA_3$  accepts the language  $(r_1+r_2)$ .

Then there is a  $FA_4$  accepts the language  $r_1r_2$ .

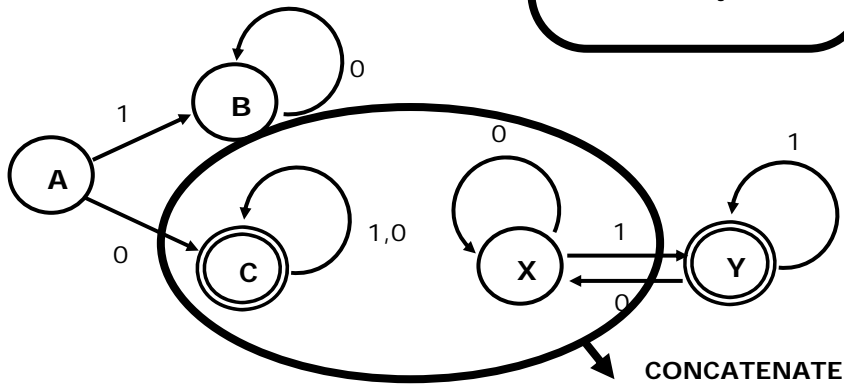
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Kleene's theorem, 1956

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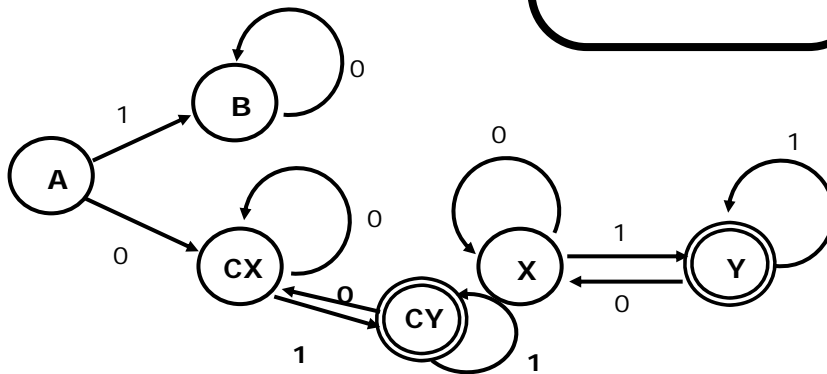
KLEENE's THEOREM

# UNIFICATION

Kleene's theorem, 1956

ทฤษฎีบท

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## KLEENE'S THEOREM

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Kleene's theorem, 1956

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วิธีพิสูจน์

Let  $FA_1$  accepts the language defined by regular expression  $r_1$ ,  
 $FA_2$  accepts the language defined by regular expression  $r_2$ .  
Then there is a  $FA_3$  accepts the language  $(r_1+r_2)$ .

Then there is a  $FA_4$  accepts the language  $r_1r_2$ .

Then there is a  $FA_5$  accepts the language  $r_1^*$ .