8

NONDETERMINISTIC

FINITE

STATE MACHINE

NONDETERMINISTIC FA

วัตถุประสงค์

We introduce a conceptual machine that occurs in practice more frequently than the transition graph.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

นิยาม

Michael Oser Rabin & Dana Scott, 1959

A nondeterministic finite state machine (NFA) is a transition graph with

- a unique start state
- each of its edge labels is a single alphabet.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

นิยาม

Michael Oser Rabin & Dana Scott, 1959

A nondeterministic finite automaton (abbreviated NFA) is a 5-tuple $(Q, \Sigma, q_0, \delta, A)$ where

- Q means a finite set of states.
- Σ is a finite input alphabet.
- $q_0 \in Q$ named Initial state.
- $A \subseteq Q$, A is the set of all accepted states.
- δ is a function from $Q \times \Sigma$ to P(Q) where P(Q) is the power set of Q.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

For a NFA, M = $(Q, \Sigma, q_0, \delta, A)$ and any $p \in Q$, $\delta^*(p,\Lambda) = \{p\}$.
For any $p \in Q$ and $x = a_1a_2a_3\ldots a_n \in \Sigma^*$ (with $n \geq 1$)
$\delta^*(p,x)$ is the set of all states q for which
there is a sequence of states $p=p_0, p_1p_2\ldots p_{n-1}, p_n=q$ satisfying

- $p_i \in \delta(p_{i-1},a_i)$ for each i with $1 \leq i \leq n$.

$$\delta^*(p,ya_n) = \bigcup_{\text{all } r \in \delta^*(p,y)} \delta(r,a_n)$$

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

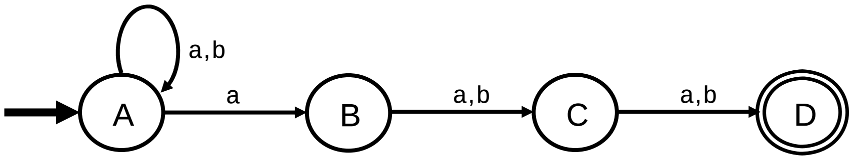
ACCEPTANCE BY A NFA

The set of all strings that leave the NFA in the final state is called the language defined by the NFA or the language associated with this machine.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

EXAMPLE

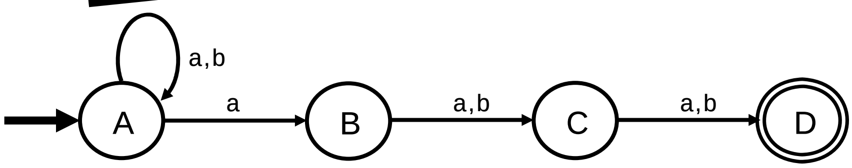


δ*(A,aa)

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

EXAMPLE



δ*(A,aba)

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

WITH Λ-TRANSITION

นิยาม

A nondeterministic finite automaton with Λ-transition (NFA-Λ) is a 5-tuple $(Q, \Sigma, q_0, \delta, A)$ where

- Q means a finite set of states.
- Σ is a finite input alphabet.
- $q_0 \in Q$ named Initial state.
- $A \subseteq Q$, A is the set of all accepted states.
- δ is a function from $Q \times (\Sigma \cup \{\Lambda\})$ to $P(Q)$ where $P(Q)$ is the power set of Q.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

WITH Λ-TRANSITION

For an NFA-Λ $M=(Q, \Sigma, q_0, \delta, A)$ states p and $q \in Q$ and a string $x = a_1a_2a_3\ldots a_n \in \Sigma^*$, we will say M moves from p to q by a sequence of transitions corresponding to x if

there exist an integer $m \geq n$, a sequence $b_1b_2b_3\ldots b_m \in \Sigma \cup \{\wedge\}$ satisfying $x = b_1b_2b_3\ldots b_m$ and
a sequence of states $p = p_0, p_1, p_2, \ldots, p_m=q$ so that
for each i, $1 \leq i \leq m$, $p_i \in \delta(p_{i-1},b_i)$.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด WITH Λ-TRANSITION

there exist an integer $m \geq n$, a sequence $b_1 b_2 b_3 \ldots b_m \in \Sigma \cup \{\wedge\}$ satisfying $x = b_1 b_2 b_3 \ldots b_m$ and
a sequence of states $p = p_0, p_1, p_2, \ldots, p_m = q$ so that
for each i, $1 \leq i \leq m$, $p_i \in \delta(p_{i-1}, b_i)$.

For $x \in \Sigma^*$ and $p \in Q$, $\delta^*(p,x)$ is the set of all states $q \in Q$ such that there is a sequence of transitions corresponding to x by which M moves from p to q.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด WITH Λ-TRANSITION

Λ-closure

Let $M=(Q, \Sigma, q_0, \delta, A)$ be a NFA-Λ.
Let S be any subset of Q.
The Λ-closure of S is the set Λ(S) defined as follows:

- Every element of S is an element of Λ(S).
- For any $q \in \Lambda(S)$, every element of $\delta(q,\Lambda)$ is in Λ(S).
- No other elements of Q are in Λ(S).

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

WITH Λ-TRANSITION

Λ-closure

For a NFA-Λ M = $(Q, \Sigma, q_0, \delta, A)$.
The extended transition function $\delta^*: Q \times \Sigma^* \rightarrow P(Q)$ is defined as follows:
For any $q \in Q$, $\delta^*(q,\Lambda) = \Lambda(\{q\})$
For any $q \in Q$, $y \in \Sigma^*$ and $a \in \Sigma$,

$$\delta^*(q,ya) = \Lambda\left(\cup_{r \in \delta^*(q,y)} \delta(r,a) \right)$$

A string x is accepted by M if $\delta^*(q_0,x) \cap A \neq \varnothing$.
The language recognized by M is the set L(M) of all strings accepted by M.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

THEOREM

ทฤษฎีบท

For every nondeterministic finite state machine, there is some finite state machine accepts exactly the same language.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

THEOREM

ทฤษฎีบท

For every nondeterministic finite state machine with Λ, there is some finite state machine accepts exactly the same language.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

THEOREM

ทฤษฎีบท

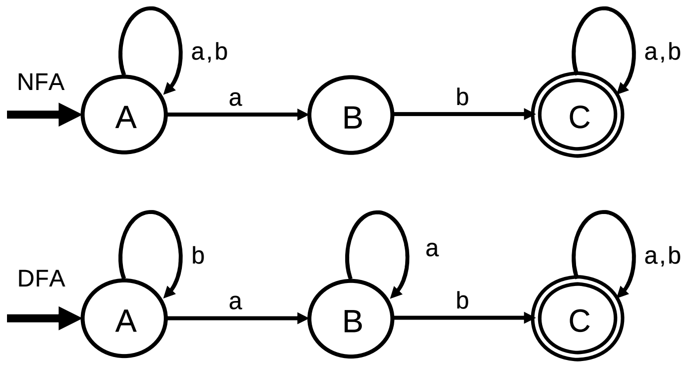
For any alphabet Σ and
any $L \subseteq \Sigma^*$,
these three statements are equivalent:

- L can be recognized by FA
- L can be recognized by NFA
- L can be recognized by NFA-Λ.

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

EXAMPLE

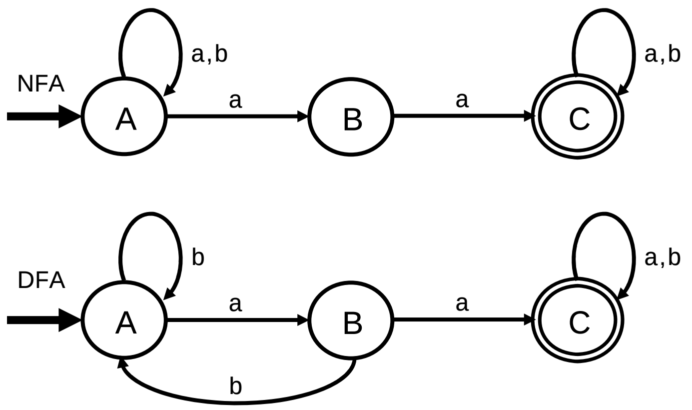


Two machines accept the same language (a+b)*ab(a+b)*

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

EXAMPLE

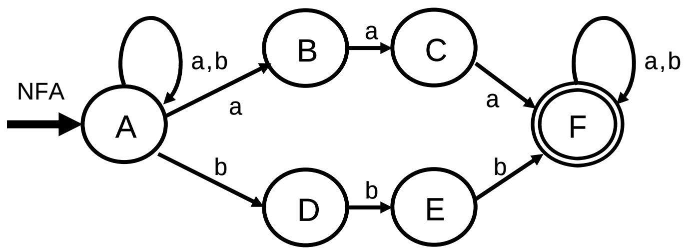


Two machines accept the same language (a+b)*aa(a+b)*

NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

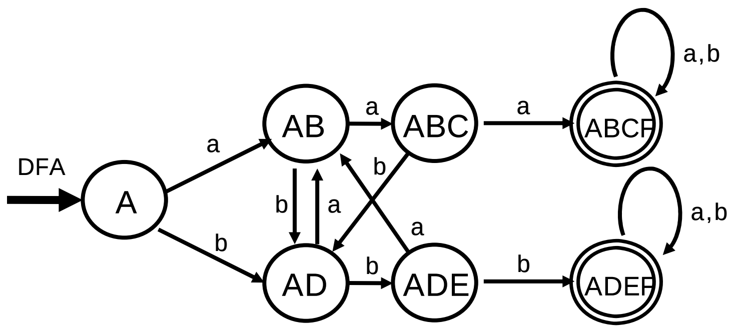
EXAMPLE



NONDETERMINISTIC FA

NONDETERMINISTIC ออโตมาตาจำกัด

EXAMPLE



Two machines accept the same language (a+b)*(aaa+bbb)(a+b)*