



Greedy algorithm

T H E O R E T I C A L M O D E L

March 29, 2005



Introduction

Typically, in optimization problems the algorithm needs to make a series of choices whose overall effect is to minimize the total cost, or maximize the total benefit, of some system. The greedy method consists of making the choices in sequence such that each individual choice is best according to some limited “short-term” criterion that is not too expensive to evaluate.



Optimal binary search tree

Create an optimal binary search tree

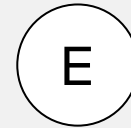
DATA	Probability
A	0.08
B	0.22
C	0.20
D	0.18
E	0.25
F	0.05
G	0.02



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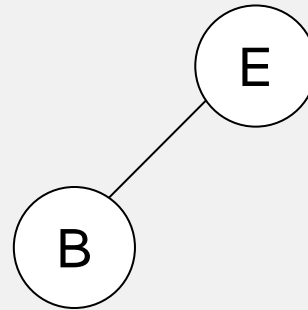




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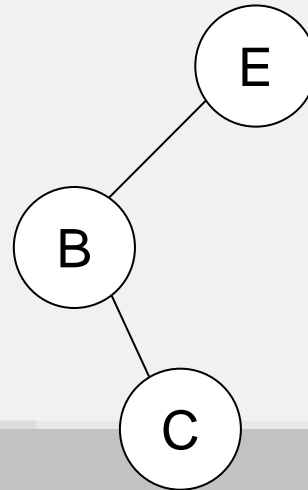




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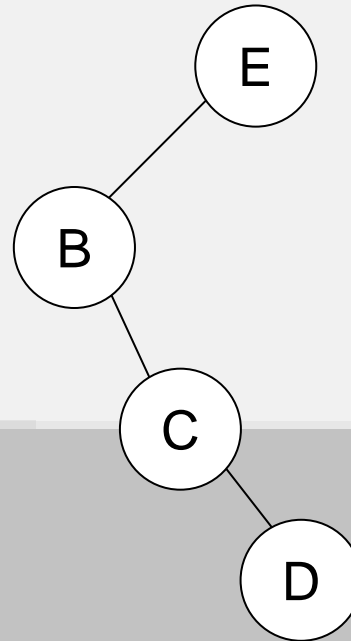




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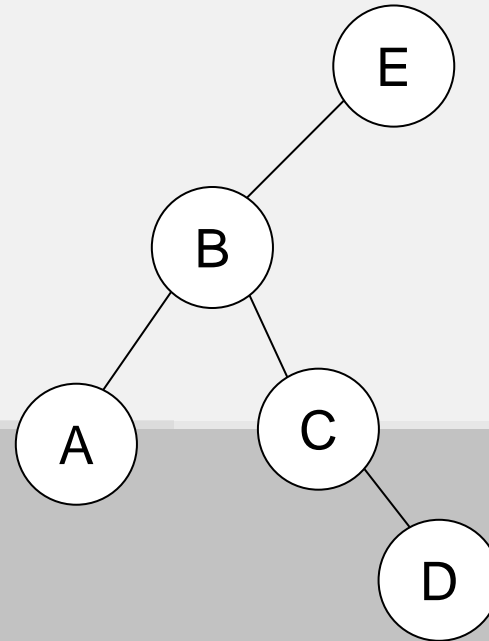




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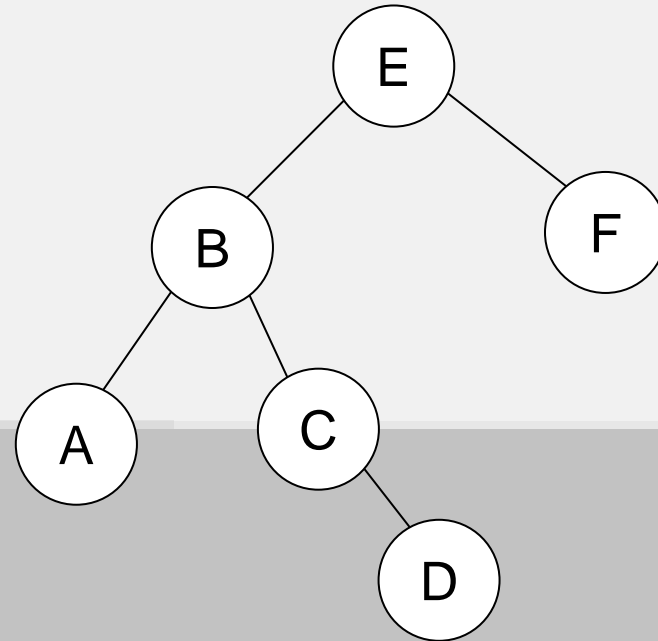




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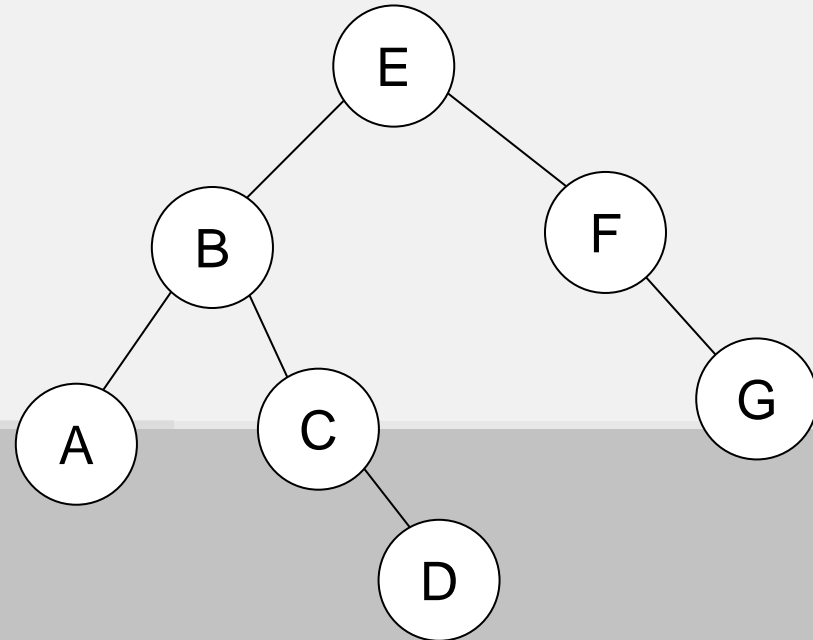




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Average-time for searching **2.41**



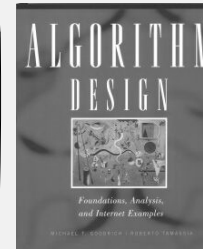
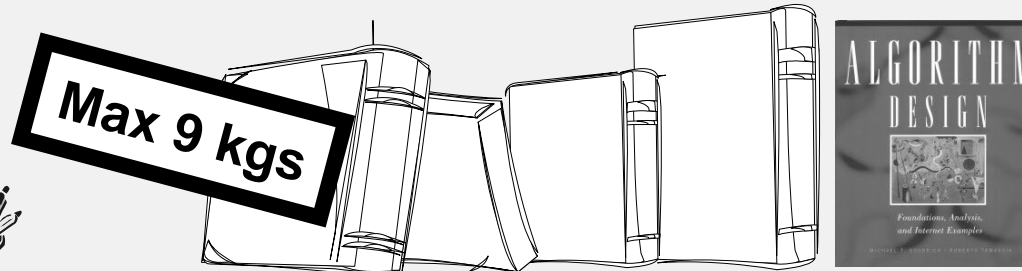
Definition

An algorithm that always takes the best immediate, or local, solution while finding an answer. Greedy algorithms find the overall, or globally, optimal solution for some optimization problem, but may find less-than-optimal solutions for some instances of other problems.



0/1 Knapsack problem

Choose items with maximum total benefit but with some limitation.



weight	4 kgs	2 kgs	2 kgs	6 kgs	2 kgs
value	B200	B30	B60	B240	B800

Greedy algorithm

Value/kg	B50	B15	B30	B40	B400
	2	5	4	3	1

Solution : B1060

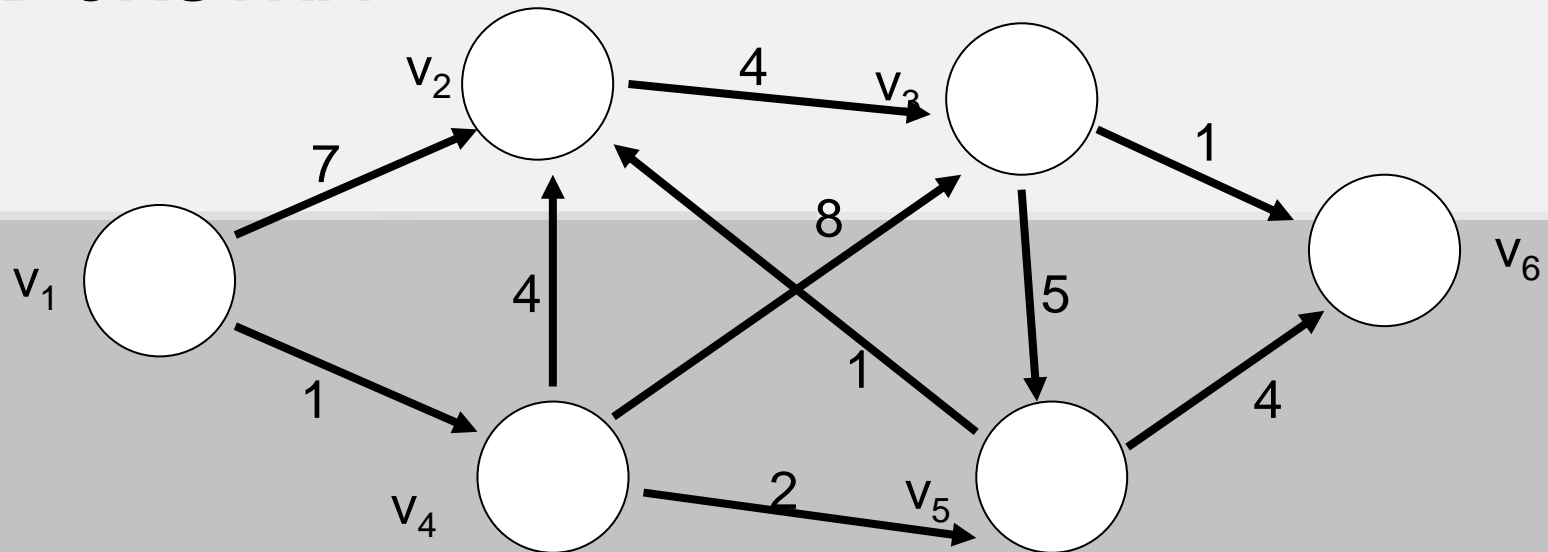


Shortest path

SINGLE-SOURCE

Given a nonnegative weight graph $G = (V, E)$
Find a shortest path from a in V to other vertices in V .

DIJKSTRA



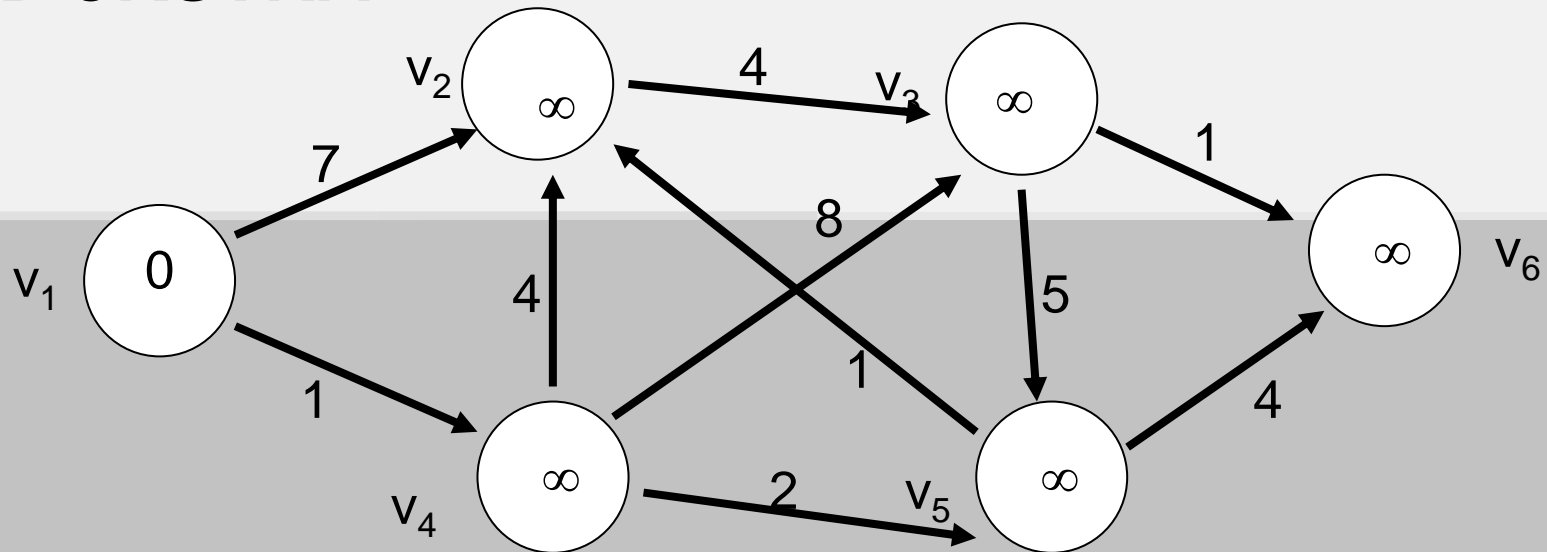


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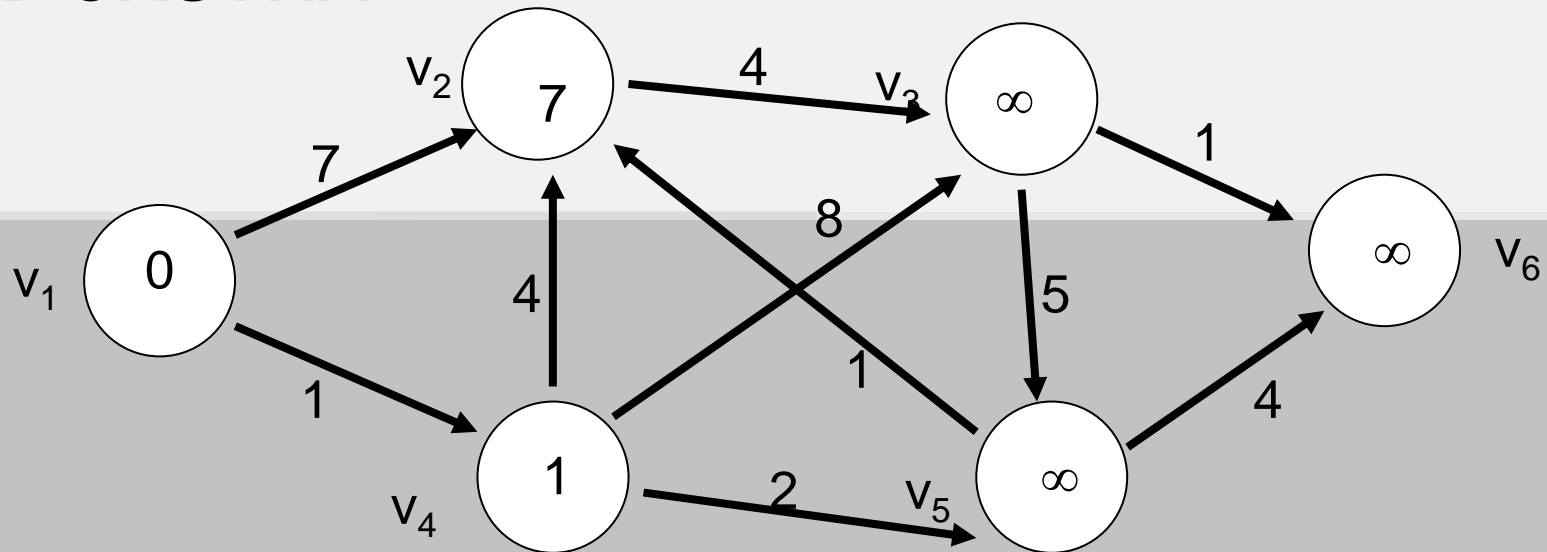


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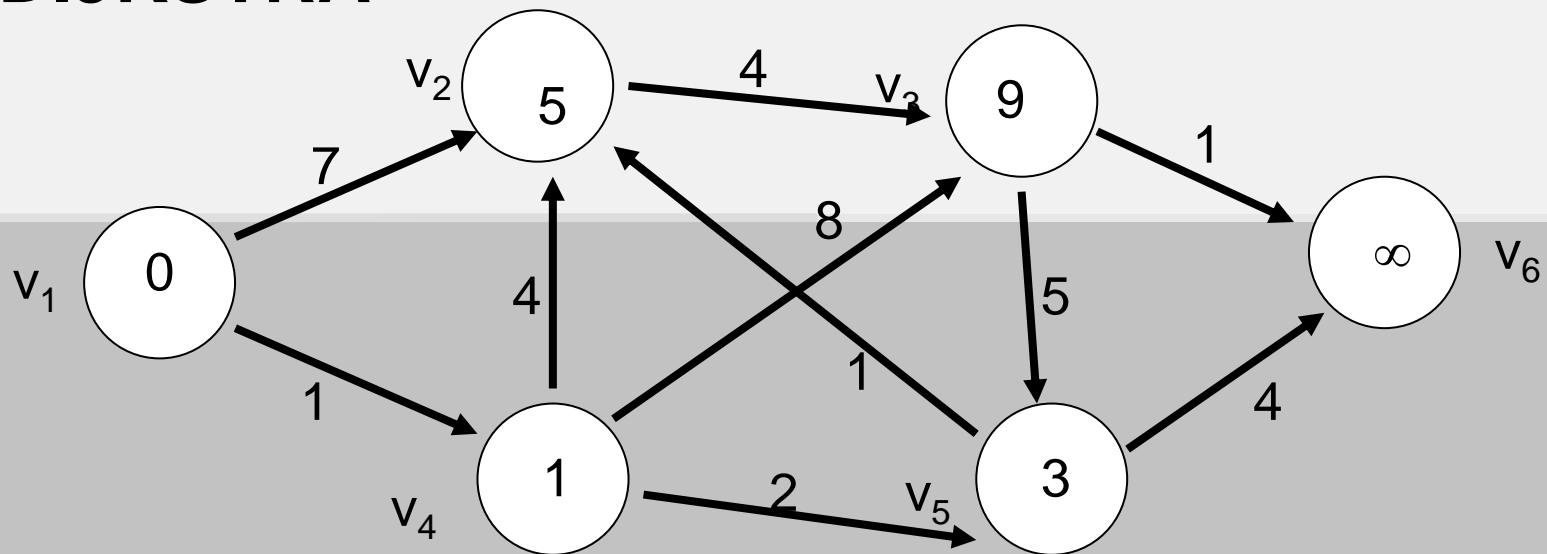


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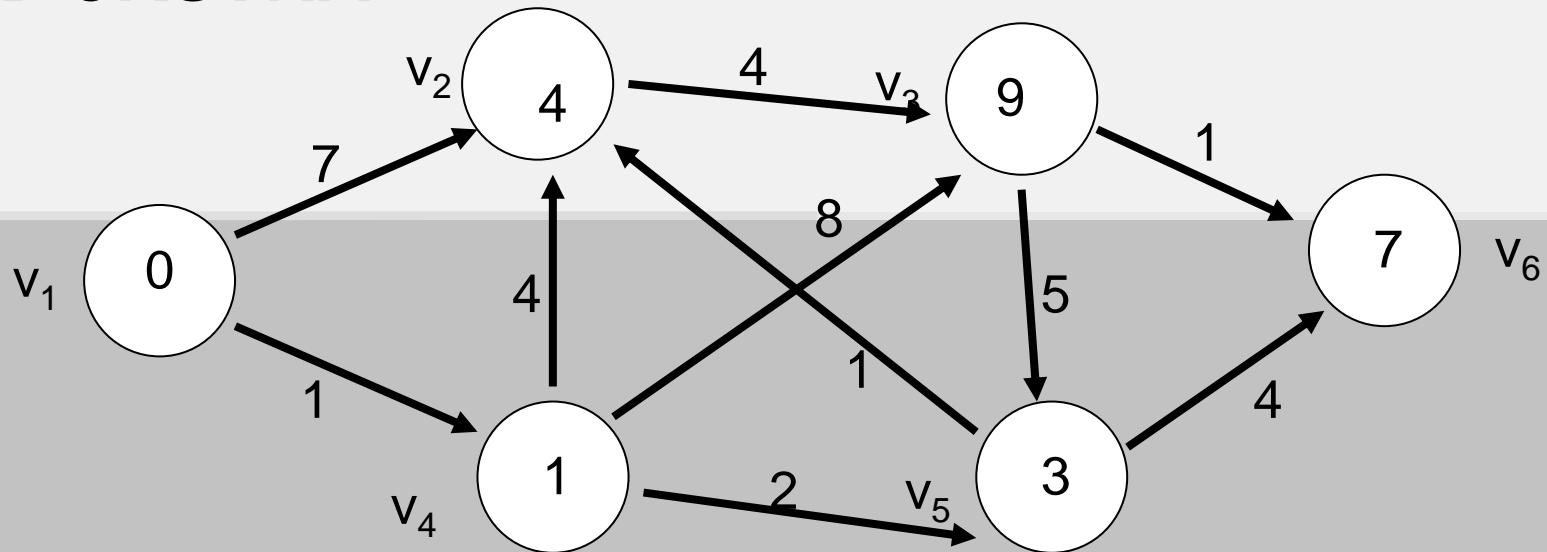


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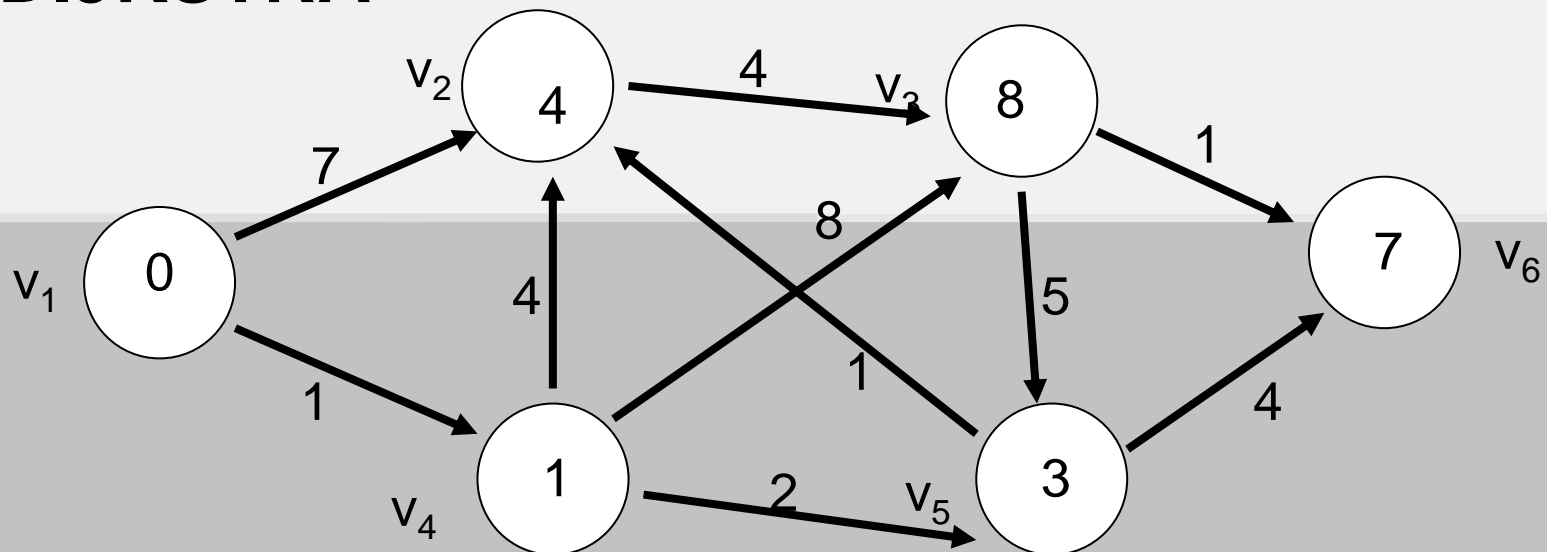


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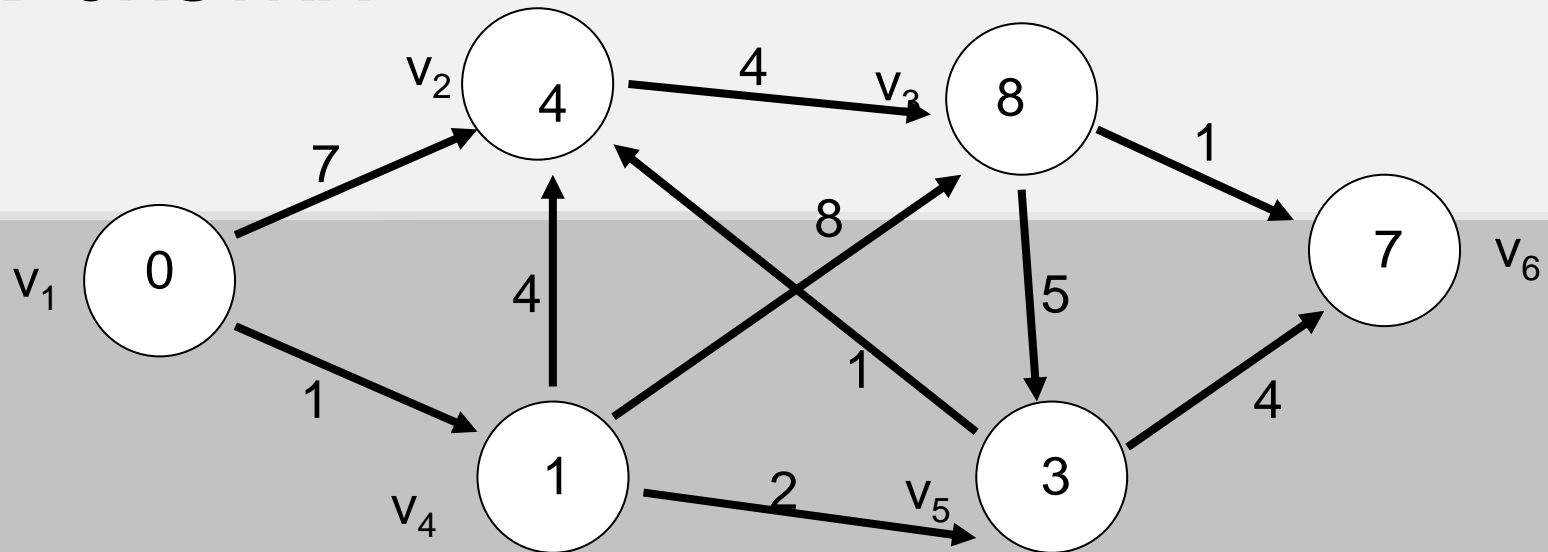


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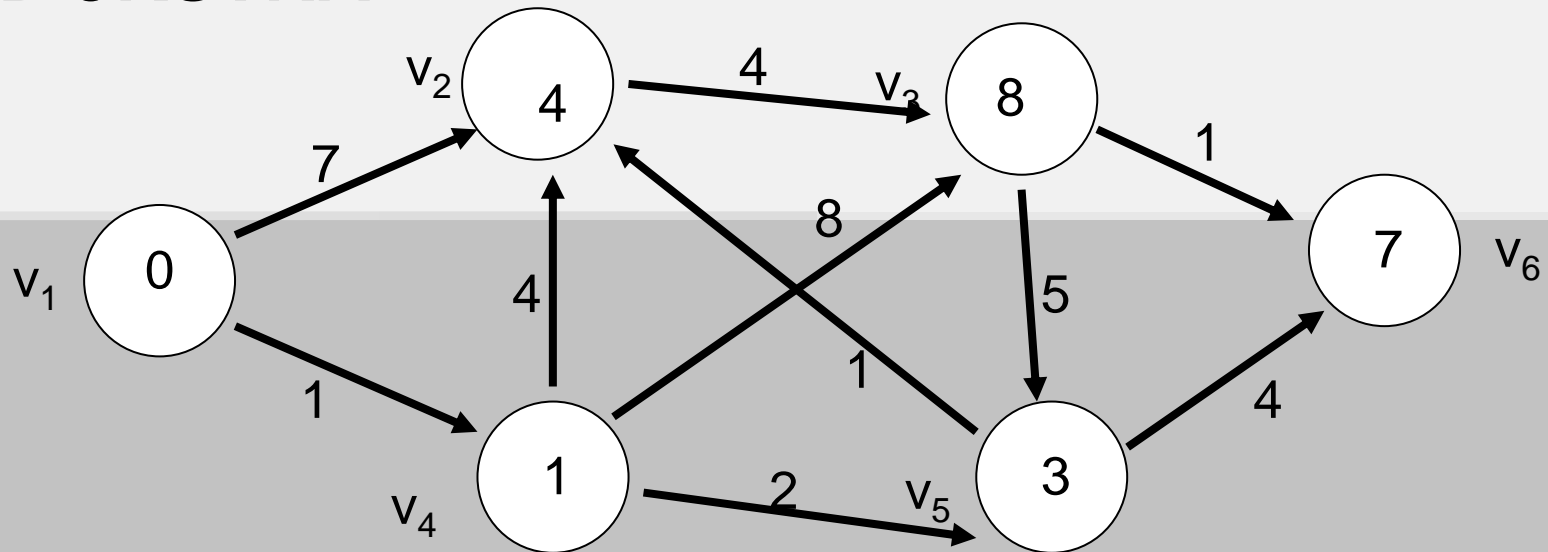


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DIJKSTRA





Activity- selection

Given a set of activities $\{ a_1, a_2, a_3, \dots, a_n \}$.

Let s_j and f_j be a started and finished time for the activity a_j .

a_j	A	B	C	D	E	F
s_j	0	3	3	5	1	5
f_j	6	5	8	9	4	7

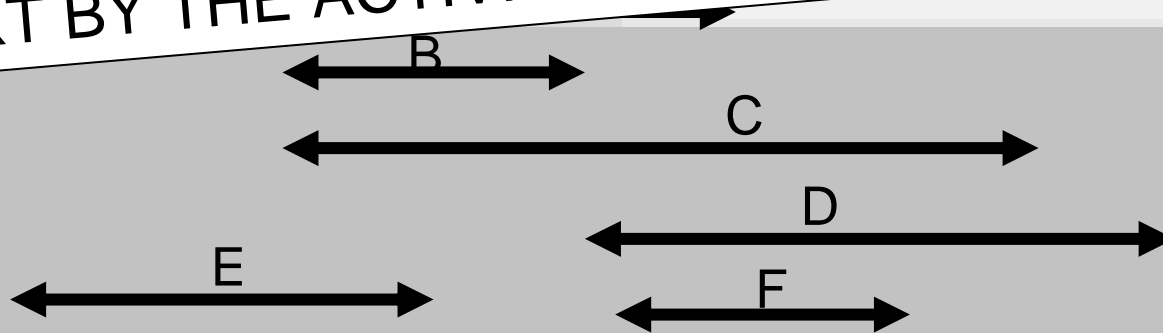
Select the maximum number of activities.



Activity-selection

a_j	A	B	C	D	E	F
s_j	0	3	3	5	1	5
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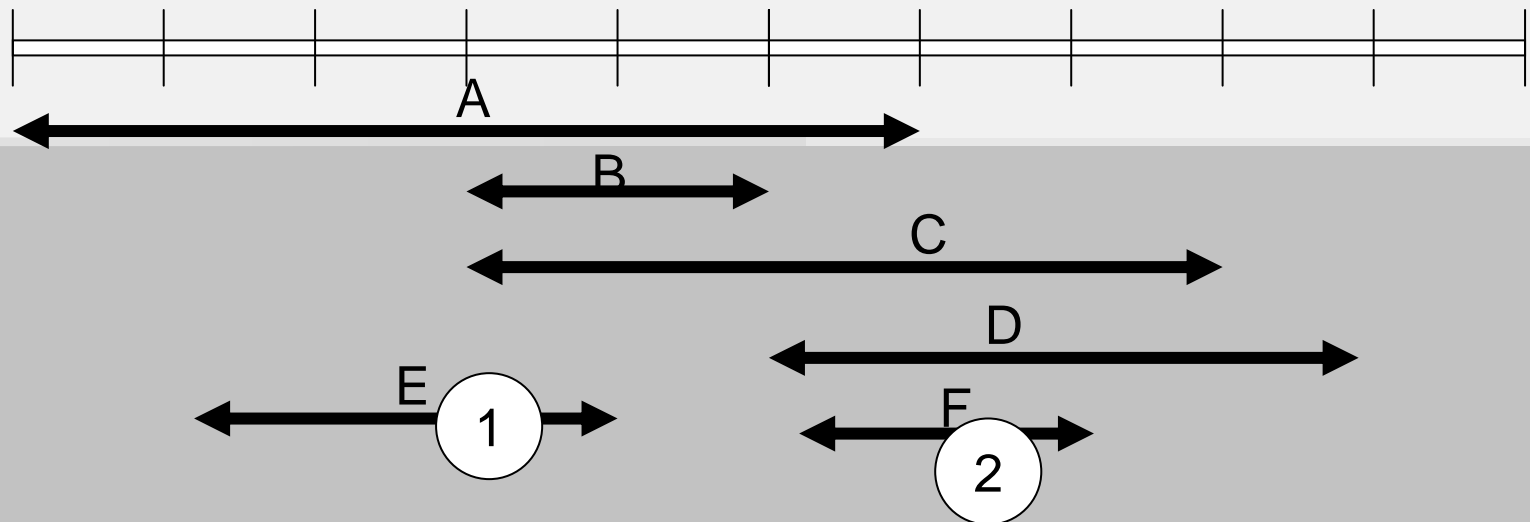
START BY THE ACTIVITY THAT FINISHES FIRSTLY.





Activity- selection

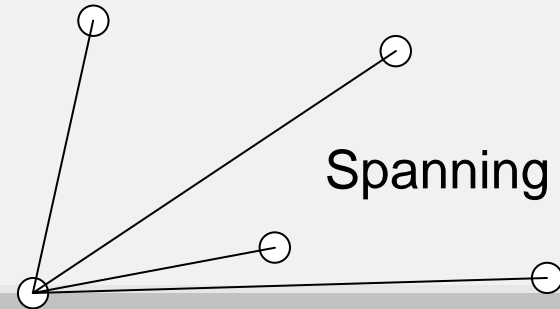
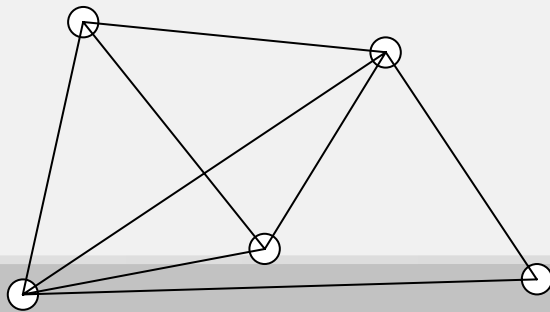
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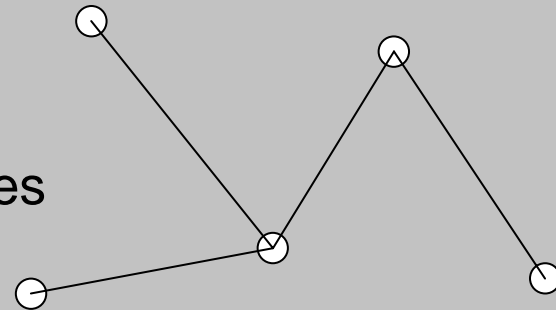
Spanning Tree

A sub-graph S of a graph G is a spanning tree if S contains all vertices of G and S is also a tree.



Spanning trees

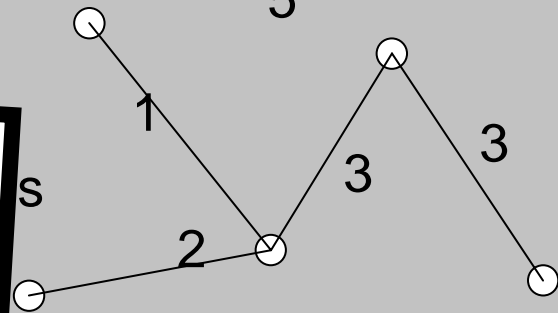
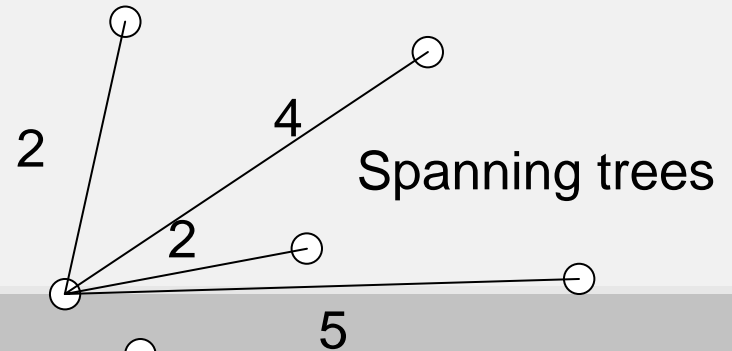
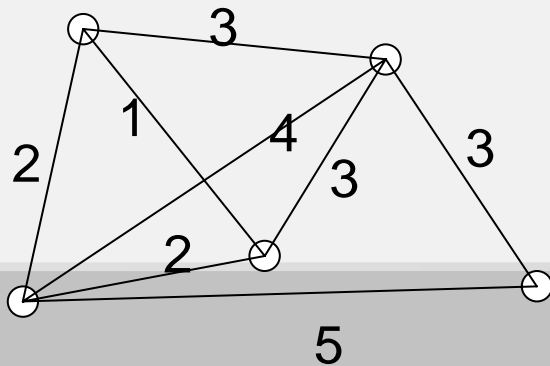
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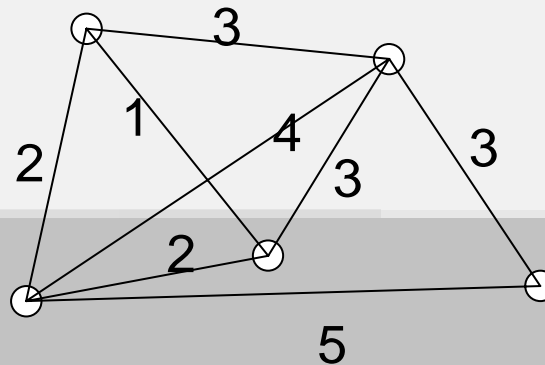
MINIMUM SPANNING TREE
minimum weight tree



Minimum spanning tree

A spanning tree with the minimum weight.

Kruskal's algorithm

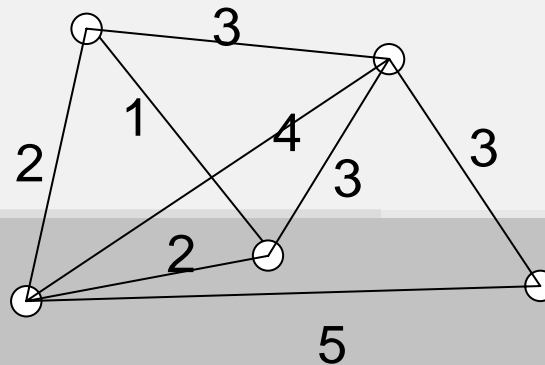




Minimum spanning tree

A spanning tree with the minimum weight.

Kruskal's algorithm

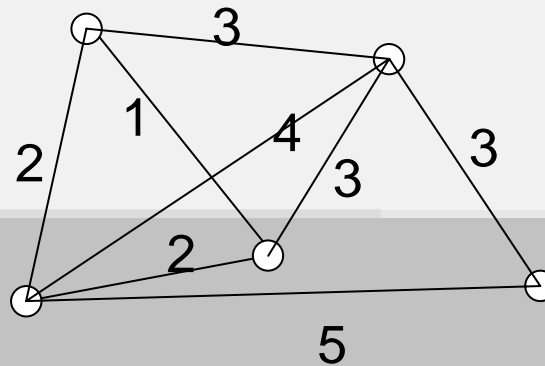




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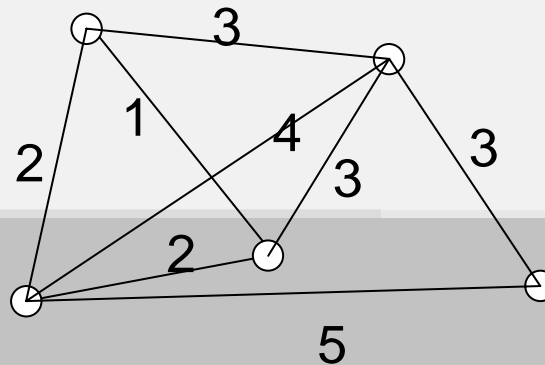




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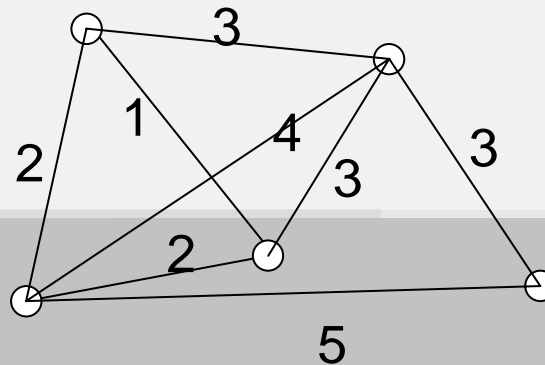




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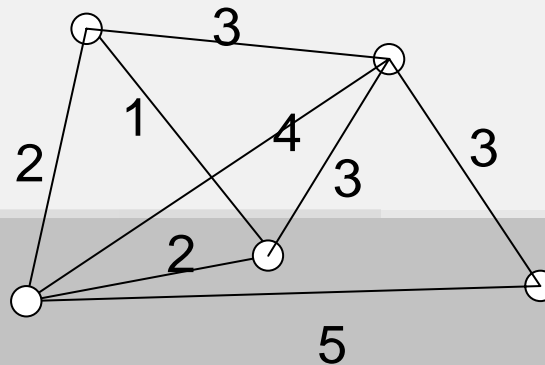




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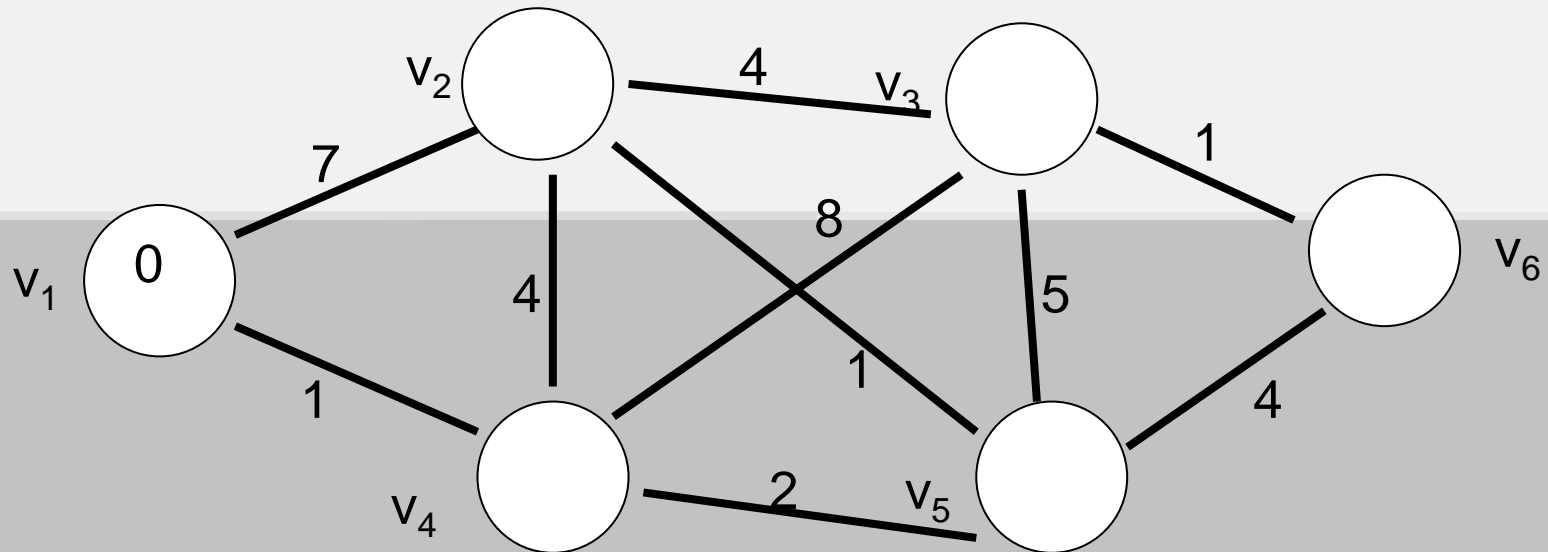




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Prim's algorithm

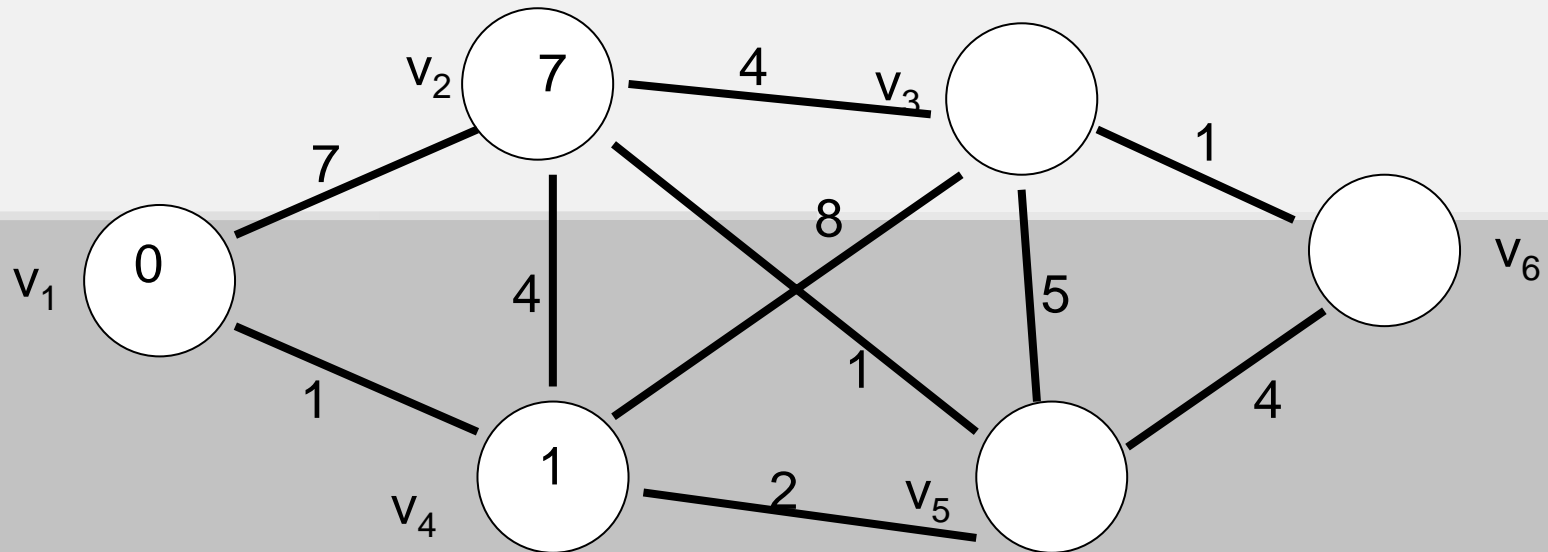




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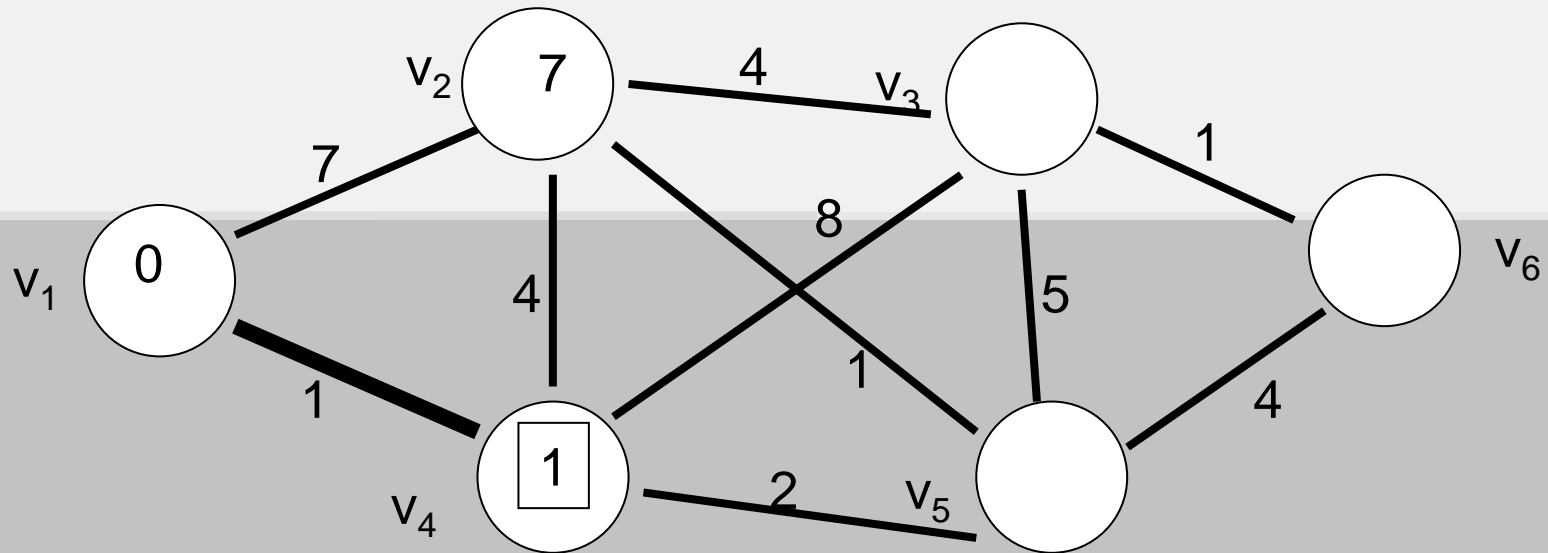




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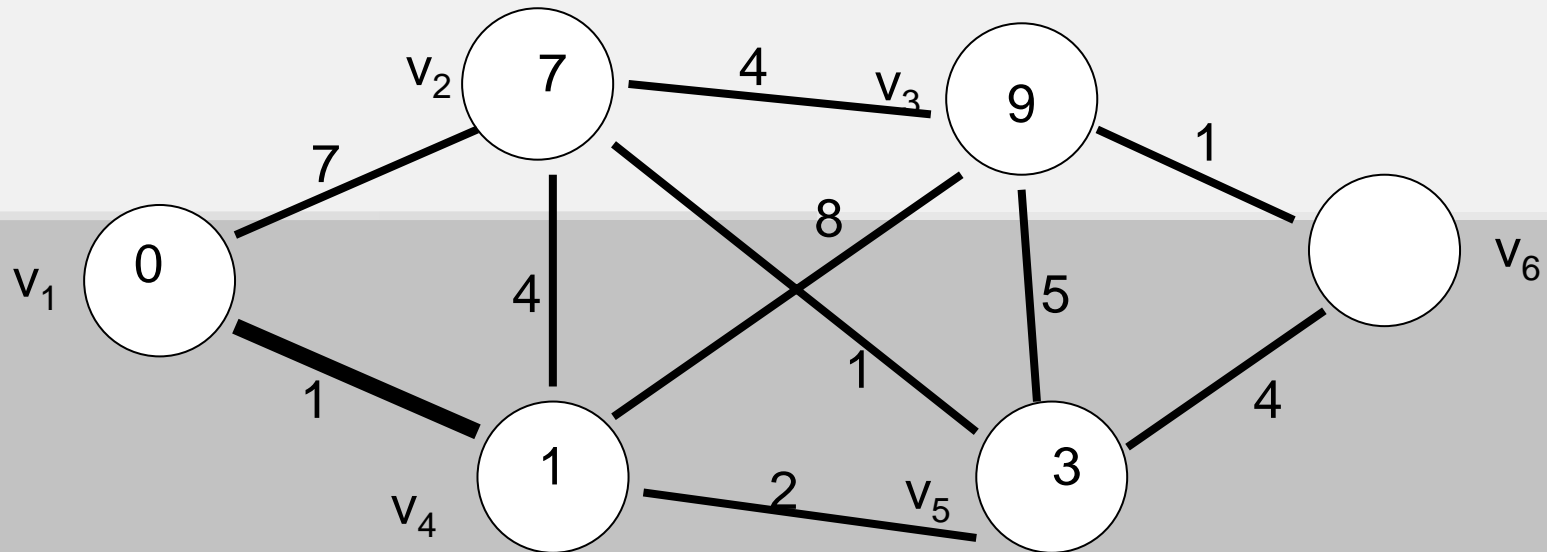




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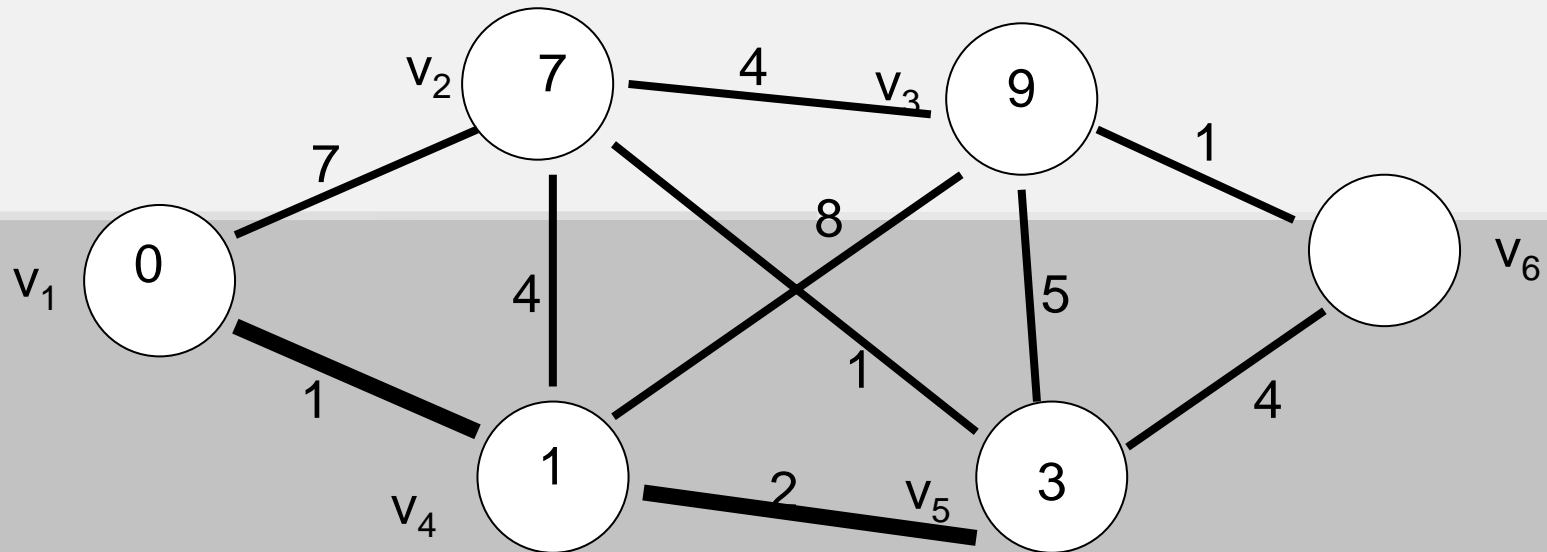




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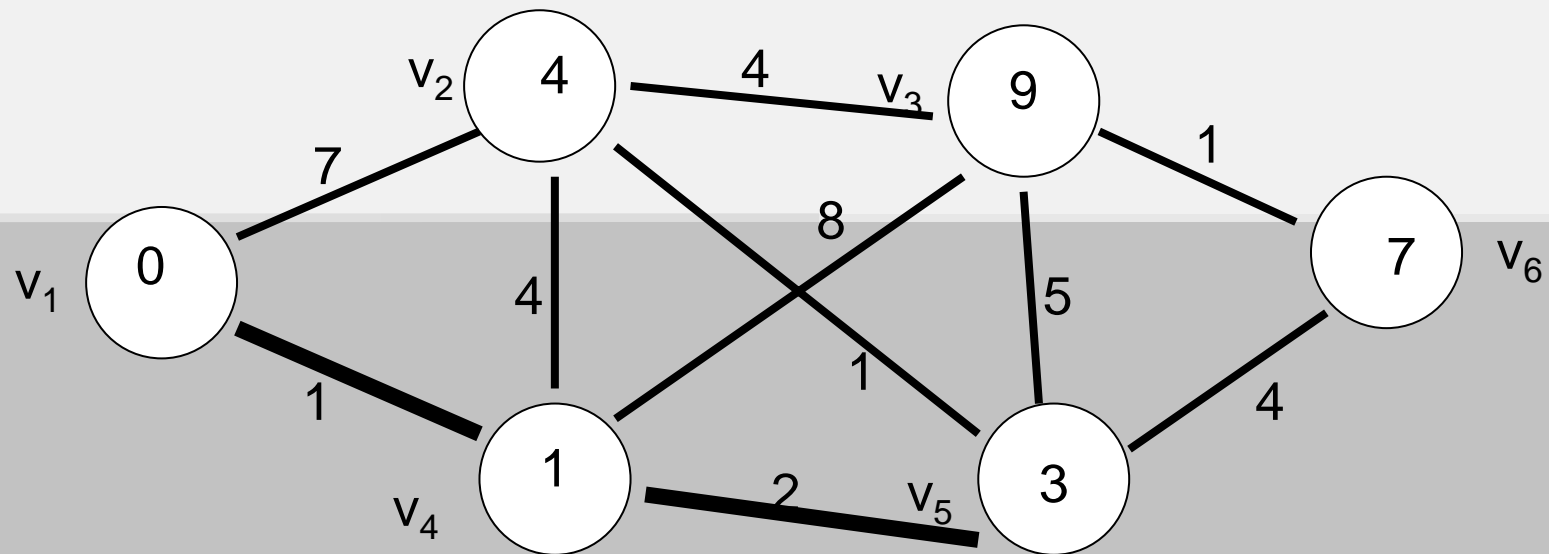




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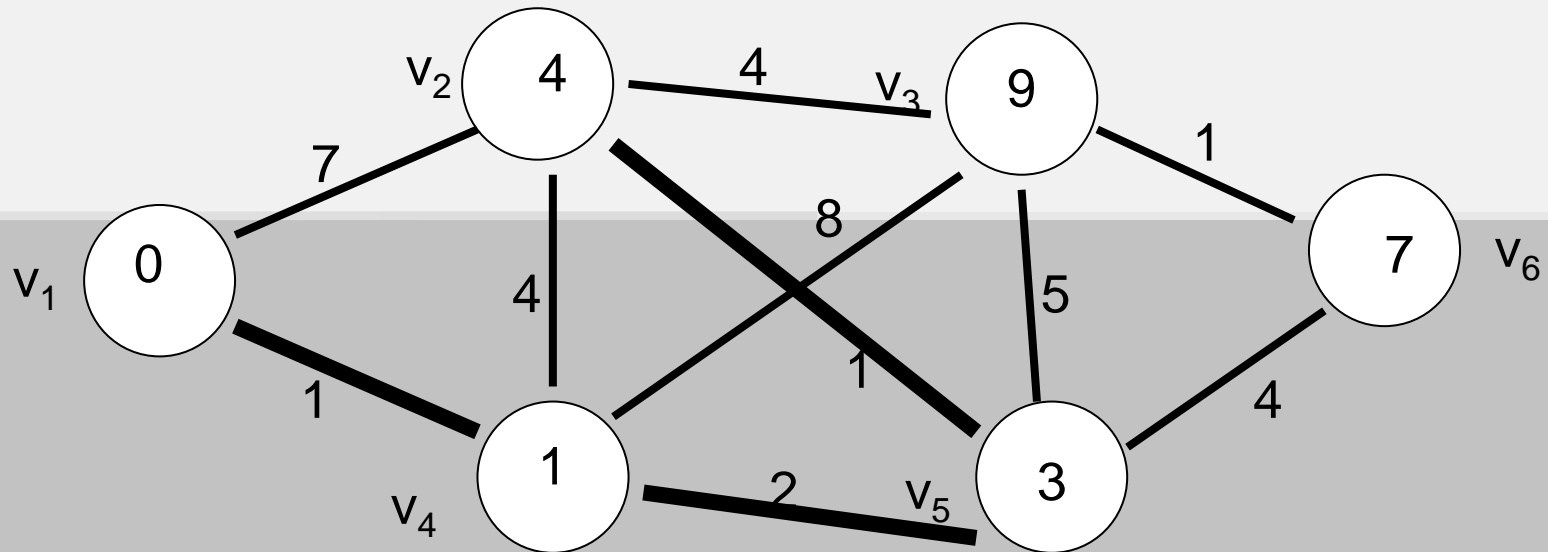




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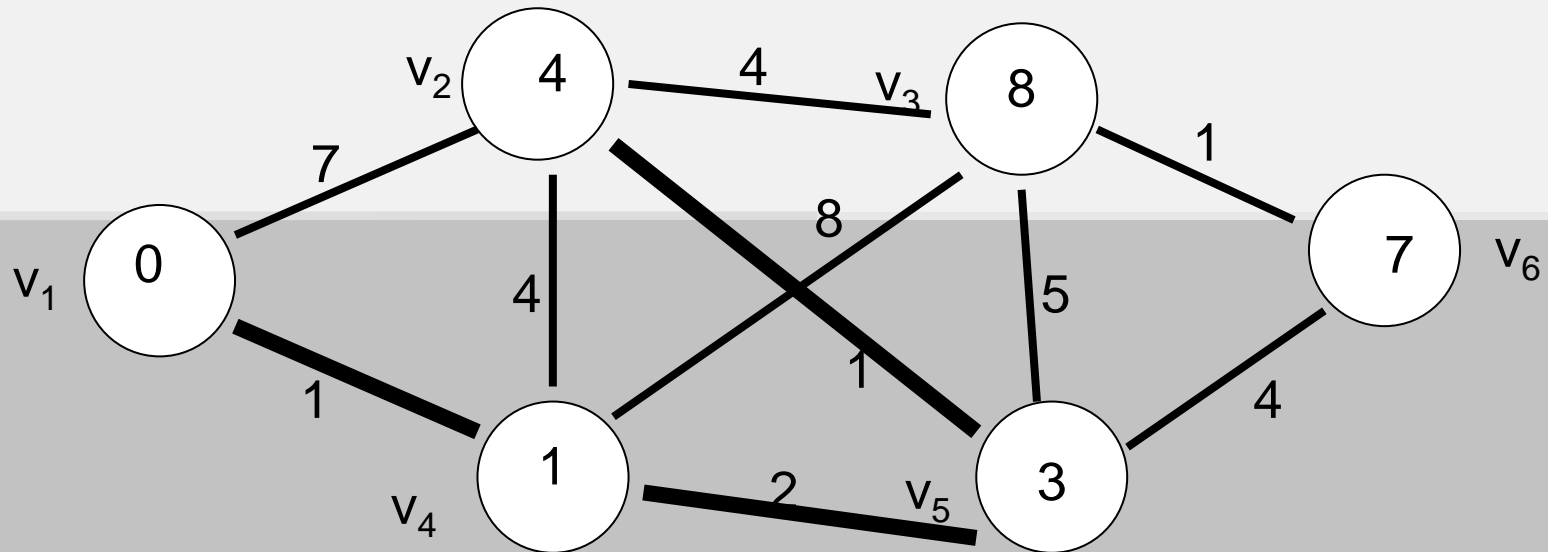




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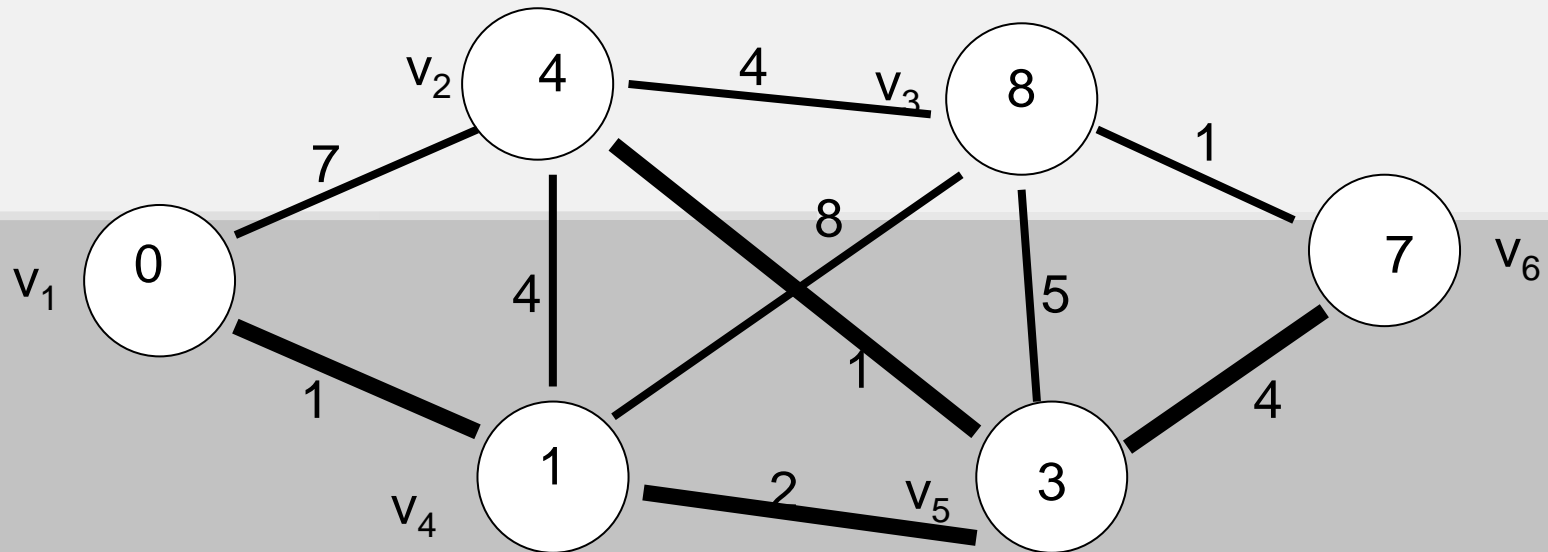




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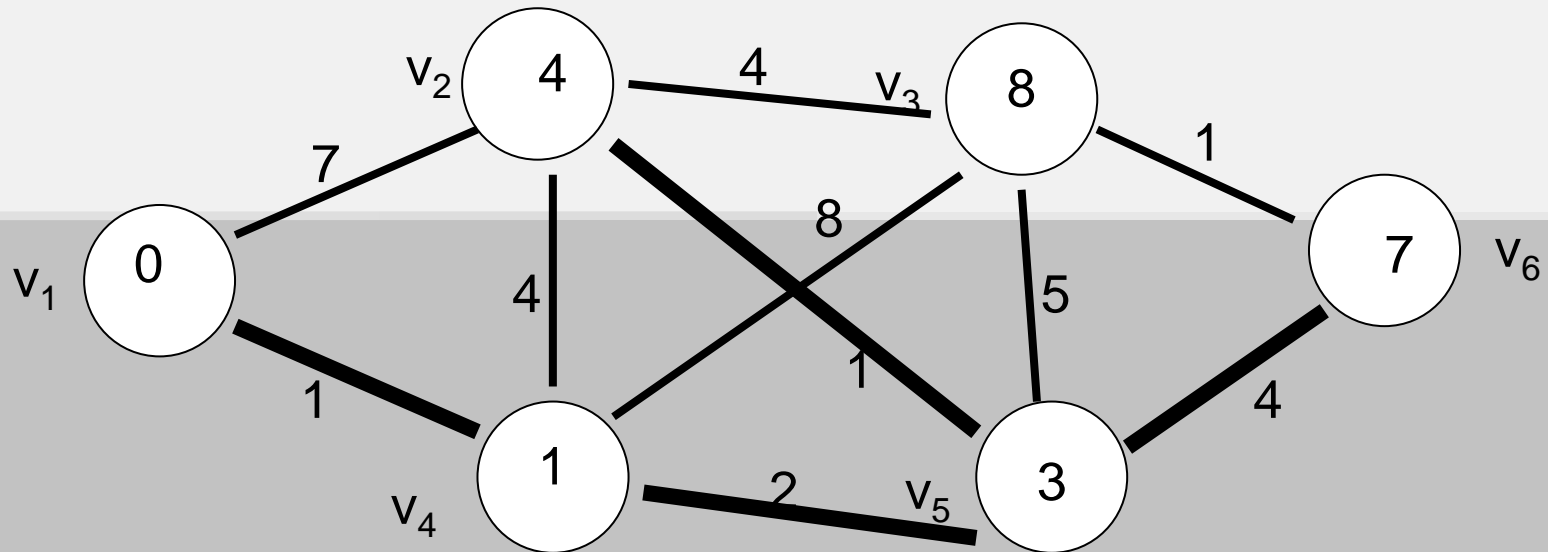




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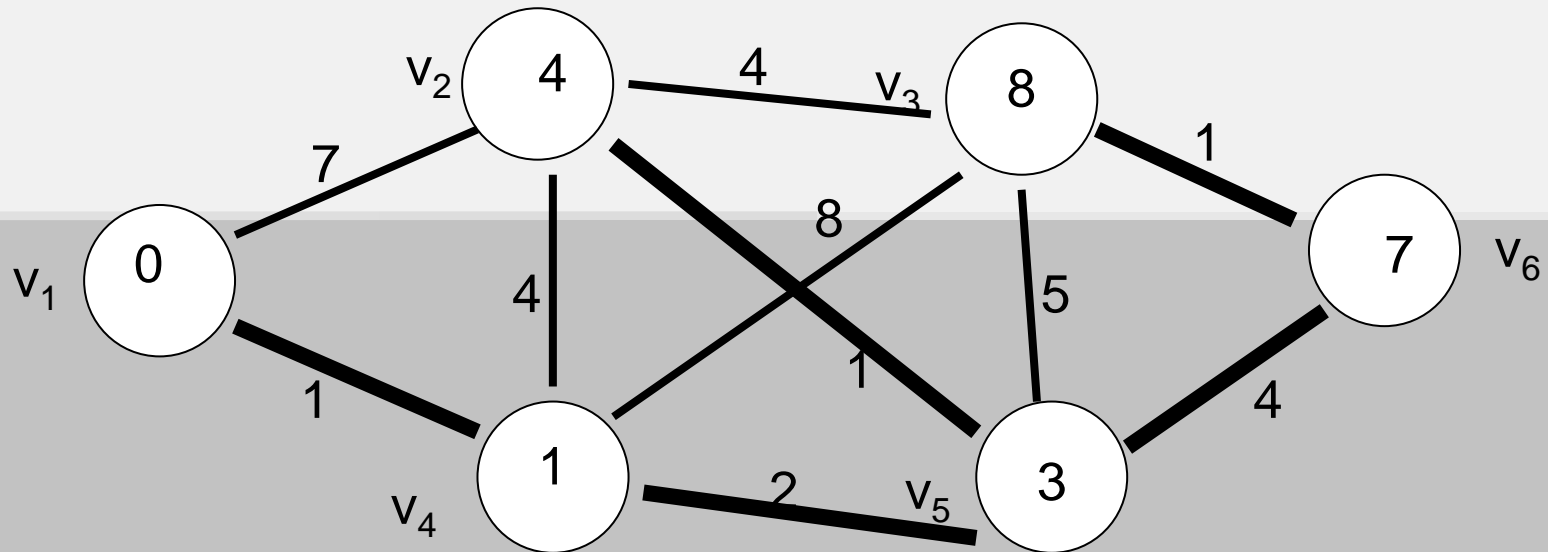




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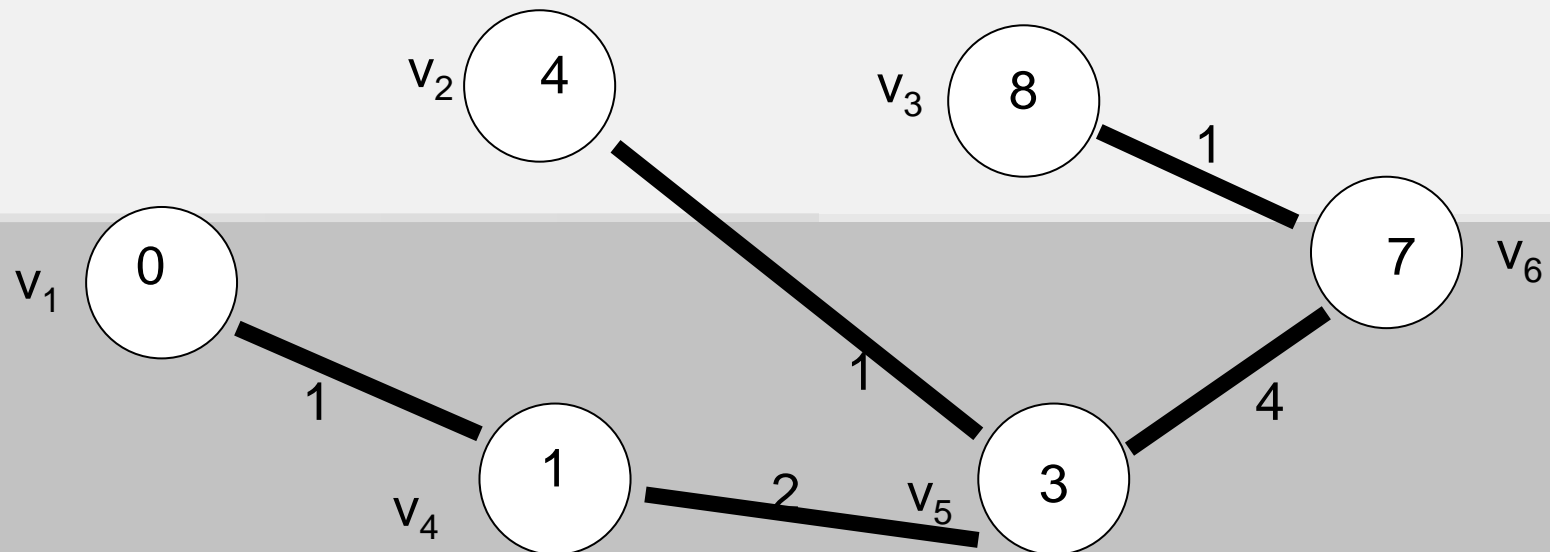




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Huffman code

Find an algorithm for encoding these characters.

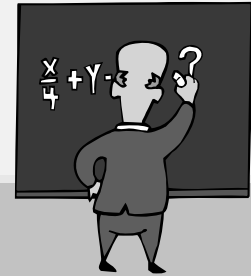
Characters	frequency
A	40
B	14
C	15
D	8
E	2
F	21
G	18



Problems

Three groups of complexity problems

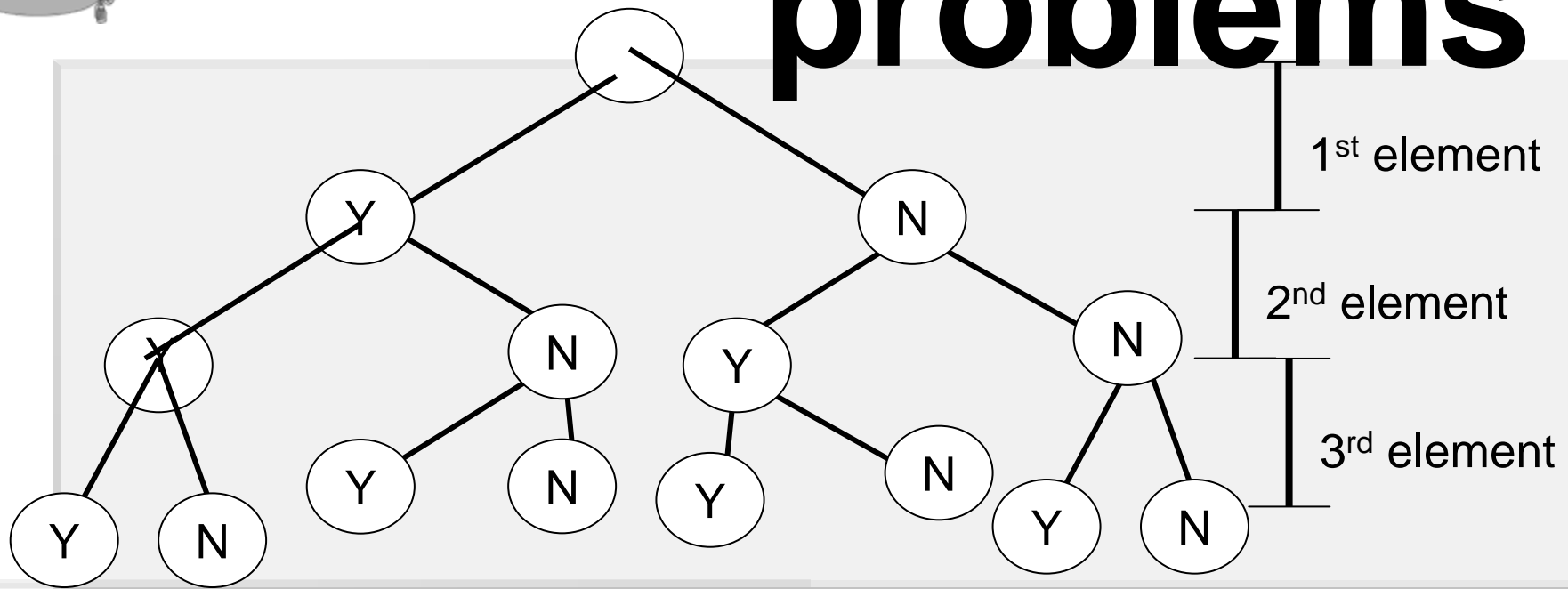
- **S**ubset problems (2^n)
- **P**ermutation problems ($n!$)
- **P**artition problems ($n!$)



State-space graph / tree



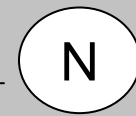
Subset problems



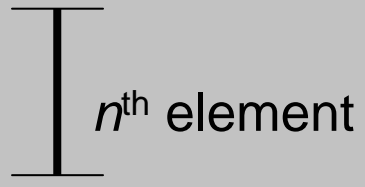
Number of nodes = $2^{n+1} - 1$



All yes



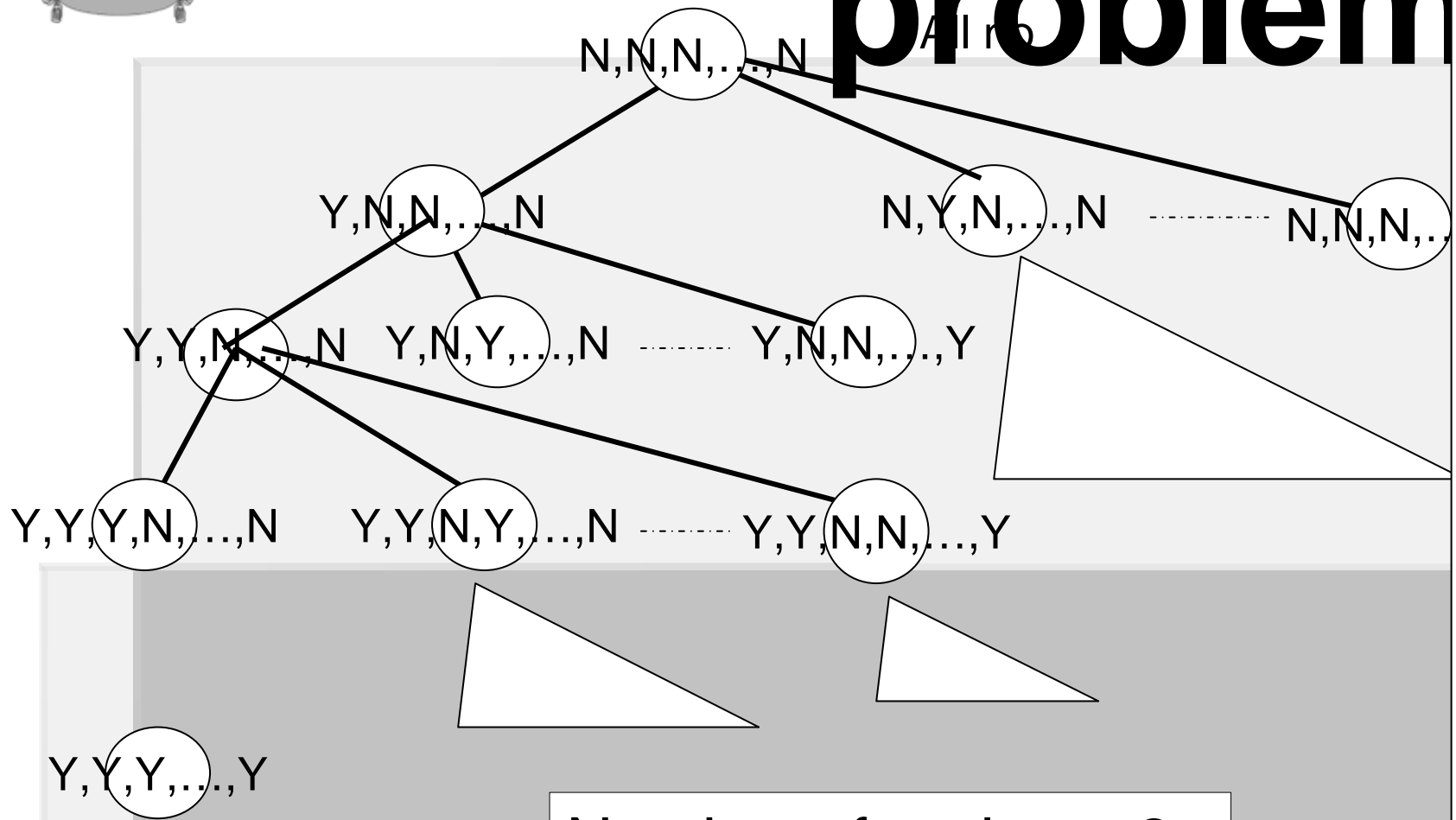
All no





Subsequence problem

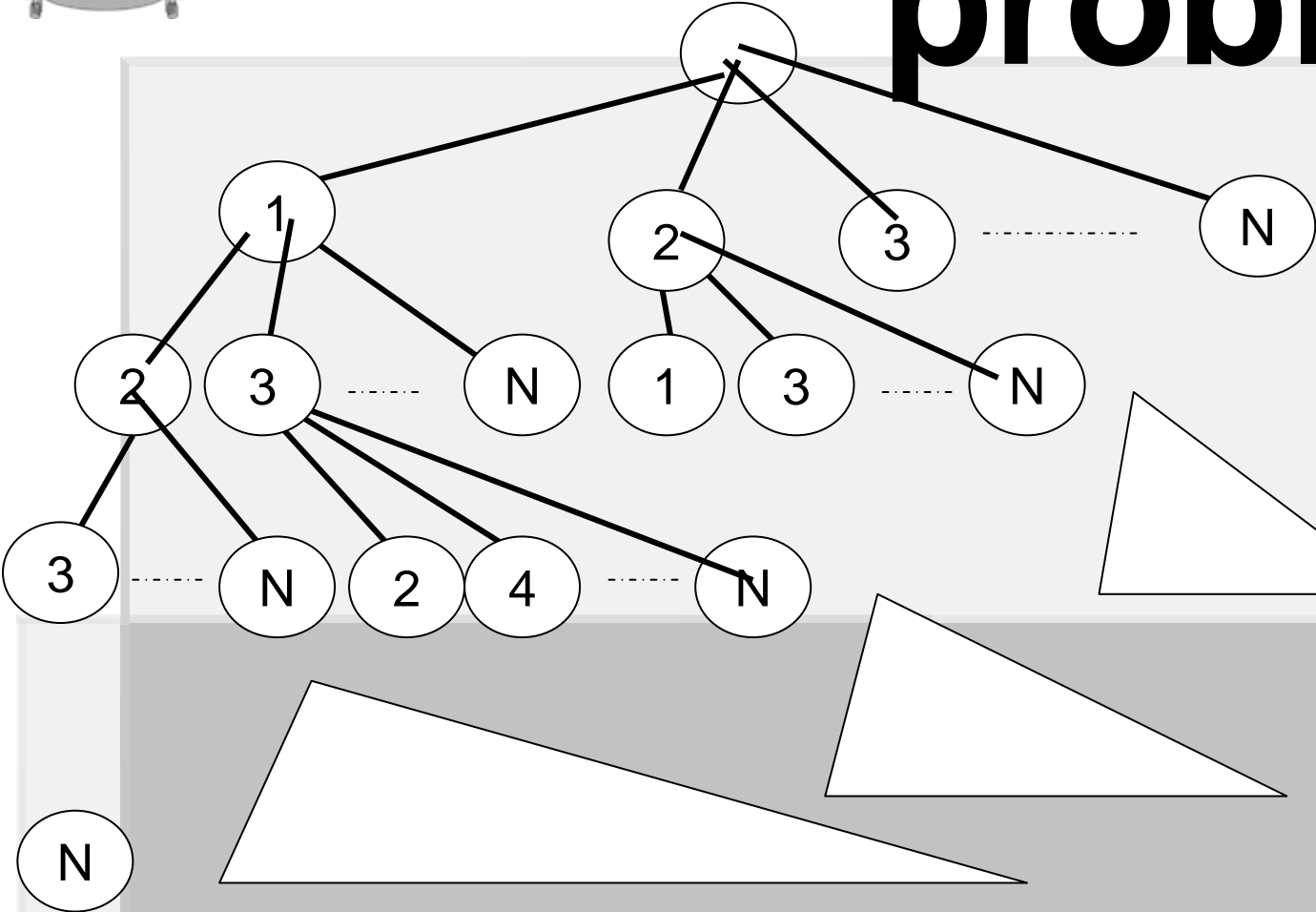
- 0000
- 1000
- 1100
- 1110
- 1111
- 1101
- 1010
- 1011
- 1001
- 0100
- 0110
- 0111
- 0101
- 0010
- 0011
- 0001



Number of nodes = 2^n

All yes

Permutation problem



- 1234
- 1243
- 1324
- 1342
- 1423
- 1432
- 2134
- 2143
- 2314
- 2341
- 2413
- 2431
- 3124
- 3142
- ...
- 4312
- 4321

1st

2nd

3rd

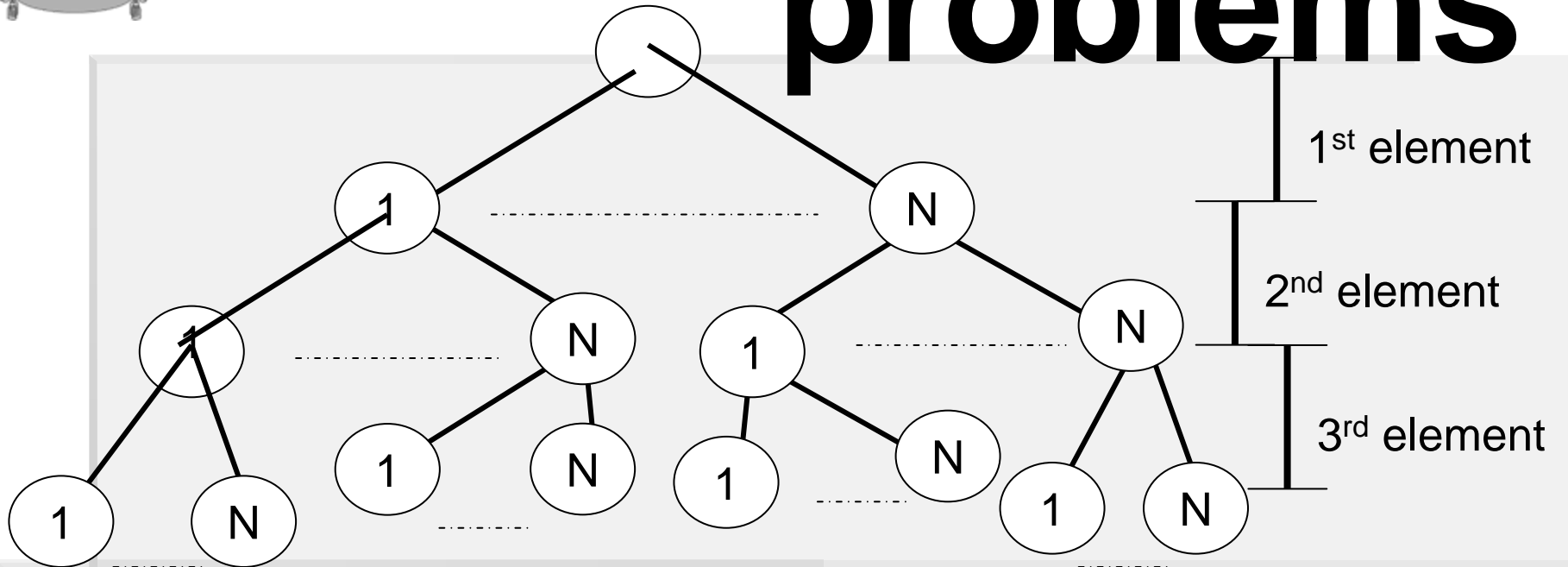
nth

n

n



Partition problems



Number of nodes = n^n



Traversal

Three possible ways

- Depth-first technique
- Breadth-first technique
- Best-first technique



Back tracking

Technique

- **D**epth-first technique
- **K**eep track and return back when it cannot be branched.



Branch & bound

	1	2	3	4
A	10	7	13	15
B	12	5	16	12
C	14	9	14	20
D	11	7	14	13