Sensorless Sorting of Two Parts in the Plane Using Programmable Force Fields

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Abstract: *A part that is placed on a massively parallel actuator array can be manipulated by the force generated by a large number of supporting actuators. At a high level of abstraction, this form of nonprehensible distributed manipulation can be modeled using programmable force fields. This paper addresses the problem of manipulating multiple parts using programmable force fields. In particular, given two convex parts in different shapes which are in unknown configurations, we introduce a sequence of force fields that isolates, recognizes, and brings each part to its target configuration without any use of a sensor. The novelty of the proposed approach lies in the part isolation stage. Our technique uses part interaction under a radial force field as a condition ensuring that both parts will be separated from each other and move into opposite halves of the plane. Once separated, each part can be processed independently and simultaneously using techniques for handling a single part.*

1 Introduction

Examples of parallel actuator arrays include, in microscale, MEMS actuator arrays [3], and in macroscale, modular distributed manipulator system [8], vibrating plates [2], and arrays of air jets [1]. A part that is placed on an actuator array can be manipulated by appropriately controlling the supporting actuators in the array. This form of nonprehensible distributed manipulation can be modeled using programmable force fields. The modeling approach represents an abstraction barrier between task-level design and hardware implementation. This analytical approach is pioneered in [3], where programmable force fields are used to represent MEMS actuator arrays. The underlying idea is that a part lying in a force field is driven toward a stable equilibrium by the resultant force and torque induced by the field at the planar contact. This basic idea allows a manipulation task to be considered as a strategy for applying a sequence of fields to bring a part from one equilibrium to another until it reaches a desired configuration. In [3], it has been shown that polygonal parts can be oriented by a sequence of squeeze fields. The sequence is planned using an algorithm similar to the one in [6] for orienting polygonal parts with a sensorless parallel jaw gripper.

Another research direction attempting to apply force fields to the positioning problem aims at inventing a single force field that can induce a unique stable equilibrium for a given part. Such a field would be able to orient any part in one step without any sensor or any sequencing control. Along this avenue, the elliptical force field that induces two stable equilibria was introduced in [7]. It was later presented in [5] with an existential proof confirming the conjecture in [3], namely, that there exists a combination of the unit radial field and a small constant field capable of uniquely orienting and positioning parts. Further progress was made recently in [9] with a constructive proof and a method for computing a field that induces a unique stable equilibrium for a given part. A sequel of the paper [10] also showed how to compute, for a set of distinct parts (with different shapes), a single field that can uniquely position and orient every part in the set.

Parallel actuator array devices provide a more promising potential to multiple object manipulation than traditional gripper based manipulators. Ability to simultaneously handle multiple parts is essential to many basic and useful tasks including part isolation, sorting, and assembly. However, the theory of force fields, so far, has been mostly applied to manipulation problems of a single part. In this paper, we investigate an application of force fields to a problem in multiple object manipulation. In particular, given two distinct convex parts in unknown configurations, we introduce a sequence of force fields that simultaneously isolates, recognizes, and brings each part to its target configuration without using a sensor. The novelty of the proposed approach lies in the part isolation stage. Our technique uses part interaction under a radial force field as a condition ensuring that both parts will be separated from each other and move into opposite halves of the plane. Once separated, each part can be processed independently and simultaneously using techniques for handling a single part.

The rest of the paper is organized as follows. We will begin by giving some background and necessary notations in Section 2. A strategy for simultaneous positioning two convex parts from unknown configurations will be presented in Section 3. This strategy consists of three main stages, namely, part isolation, sorting and positioning. They will be presented in detail respectively in Sections 3.1,3.2 and 3.3. We will then conclude the paper with discussion and conclusion in Section 4.

2 Background

Consider a two dimensional part with uniform mass distribution that is placed in the plane of a force field. We attach the world frame (ξ, η) to this plane.

The resultant force $\boldsymbol{F}$ and torque $\boldsymbol{M}$ exerted by the field $\boldsymbol{g}(\xi, \eta)$ on a part can be written as

$$\boldsymbol{F} = \int\int \boldsymbol{g}(\xi, \eta)\, d\xi d\eta \quad \text{and}$$

$$\boldsymbol{M} = \int\int \begin{pmatrix} \xi \\ \eta \end{pmatrix} \times \boldsymbol{g}(\xi, \eta)\, d\xi d\eta,$$

where both integrations are performed over the plane region occupied by the part. A part in the field achieves an equilibrium when the corresponding resultant force $\boldsymbol{F}$ and torque $\boldsymbol{M}$ vanish. Note that the lateral force modeling used here results in first order dynamics of the motion of parts under force fields. It is a commonly used hypothesis in part orientation with force fields [4, 3, 7].

In this paper, we deal with only two types of force fields: constant fields and radial fields. A *constant field* is a force field (see Figure 1(a)) with the same force at every point and a *radial field* (see Figure 1(b)) is a force field for which all forces point toward a single center and the magnitude of the force at a point depends only on the distance between the point and the center.

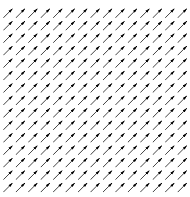
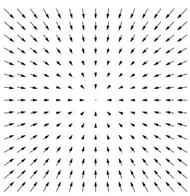
(a)  (b) 

Figure 1: Examples of (a) a constant field, and (b) a radial field.

We denote by a tuple $\langle \boldsymbol{c}, f(\lambda)\rangle$ a radial field with center $\boldsymbol{c}$ and the force at any point $\boldsymbol{p}$ be the unit force in the direction from $\boldsymbol{p}$ to $\boldsymbol{c}$, scaled by $f(\lambda)$ where λ is the distance between $\boldsymbol{p}$ and $\boldsymbol{c}$. Note that a linear radial field is a radial field for which the function f is linear in λ. We also use a pictorial representation to illustrate a radial field. Figure 2 shows an example.

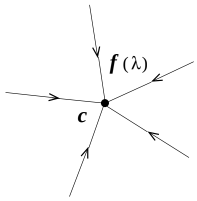


Figure 2: Pictorial representation of the radial field $\langle \boldsymbol{c}, f(\lambda)\rangle$, with $f(\lambda) \geq 0$. The arrows on the rays depict the direction of the forces.

We define the *pivot point*[1] of a part under a radial field to be a fixed point in the part's coordinate frame situated at the center of the field when an equilibrium is achieved. Note that the pivot point is unique for the unit and linear radial fields [9].

3 Handling Multiple Parts

Consider two distinct (i.e., of different shapes) planar convex parts with uniform mass distribution. Let the two parts be arbitrarily placed in the plane of force fields. We will introduce in this section a sequence of force fields that simultaneously brings both parts to their target configurations using neither sensory inputs nor the knowledge of the initial configurations. The idea is that each force field in the proposed sequence will be activated one by one. At each activation step, the parts will move from one stable equilibrium to the next, and when the entire sequence has been executed, the two parts are expected to reach the desired configurations. We assume dissipative dynamics. For a system with totally dissipative dynamics, the total energy along any trajectory always decreases. While kinetic and potential energy may be compensated at different points, no energy is added to the system; instead phenomena such as friction and viscoelasticity cause a continual loss of the total energy as time evolves. The assumption of dissipative dynamics ensures that a part under a force field will eventually stop at a stable equilibrium configuration if one exists.

The proposed sequence of force fields for moving the two convex parts to the desired configurations from the unknown ones consists of the following three sequential stages (see Figure 3):

1. Isolating parts: In this stage, the two parts which are originally in unknown configurations will be separated. Each part will move to, and stop in a different half plane. At the end of this stage, one part will lie in the top half while the other will lie in the bottom half (cannot be determined at this point which part is in which half of the plane).

[1] We borrow this term from [3] where it is defined only for the unit radial field.

2. Sorting parts: In this stage, only a specified part will move to the left half plane. We will, therefore, know which part is in which region of the plane.

3. Positioning parts: A field for orienting and positioning parts is applied independently in each half plane to move each part to a desired configuration.

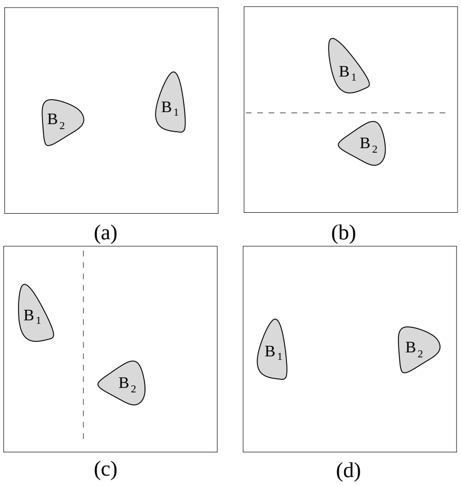


Figure 3: (a) Two parts in unknown configurations, (b) they are isolated in different half planes, (c) only a specified part is moved to the left half plane, and (d) they are positioned in desired configurations independently in each half plane.

In the following sections, we discuss each stage of the sequence in detail. In the discussion, we consider two distinct convex parts B_1 and B_2 with area A_1 and A_2 respectively. Without loss of generality, we assume $A_1 \geq A_2$. We also denote by $B_i(\boldsymbol{q}), i = 1, 2$ the plane region occupied by the part B_i when it is at the configuration $\boldsymbol{q}$.

3.1 Isolating Parts

There are two steps in isolating parts:

1. Drawing parts together: The radial field $\mathcal{J} \stackrel{\text{def}}{=} \langle \boldsymbol{o}, k\lambda \rangle$, where $k > 0$ is applied to draw the two parts together. The parts will stop in a combined stable equilibrium configuration for which the next step can ensure that the two parts will be pushed to opposite halves of the plane.

2. Pushing parts away from each others: An inverse squeeze field with its axis fixed with the plane of force field and passing through the center $\boldsymbol{o}$ is applied to push the parts to opposite half-planes separated by the field axis.

We will explain in detail how these two steps work in Sections 3.1.1 and 3.1.2. To completely understand the discussion, the following two lemmas about some properties of the field $\mathcal{J}$ are needed.

Lemma 1 *The pivot point of a part under the radial field $\mathcal{J}$ is the part's centroid.*

The proof of this lemma can be found in Appendix of the paper.

Lemma 2 *The resultant force induced by the radial field $\mathcal{J}$ on a part is $k\overrightarrow{\boldsymbol{p}\boldsymbol{o}}A$, where A denotes the part's area and $\boldsymbol{p}$ denotes the position of the centroid of the part (Figure 4).*

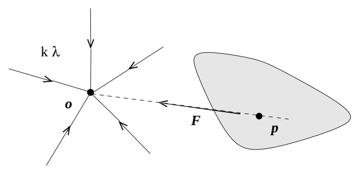


Figure 4: The resultant force $\boldsymbol{F}$ induced by the radial field $\langle \boldsymbol{o}, k\lambda \rangle$.

The proof of this lemma can also be found in Appendix of the paper.

3.1.1 Drawing Parts Together

In this step, we apply the radial field $\mathcal{J}$ to attract the two parts (from unknown configurations) toward the center of the field. The two parts will stop at a combined stable equilibrium configuration $\boldsymbol{q} = (\boldsymbol{q}_1, \boldsymbol{q}_2)$ where $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ respectively denote the configurations of the parts B_1 and B_2 when the system of the two parts is at this stable equilibrium.

For the system of B_1 and B_2, there are only two external forces[2]. They are exerted by the field $\mathcal{J}$. By Lemma 2, the field $\mathcal{J}$ exerts on B_1 the force $\boldsymbol{F}_1 = k\overrightarrow{\boldsymbol{p}_1\boldsymbol{o}}A_1$ and exerts on B_2 the force $\boldsymbol{F}_2 = k\overrightarrow{\boldsymbol{p}_2\boldsymbol{o}}A_2$ where $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are the positions of the centroids of $B_1(\boldsymbol{q}_1)$ and $B_2(\boldsymbol{q}_2)$. Clearly, the system is in equilibrium when $\boldsymbol{F}_1 = -\boldsymbol{F}_2$, or equivalently when $\overrightarrow{\boldsymbol{p}_1\boldsymbol{o}}A_1 = -\overrightarrow{\boldsymbol{p}_2\boldsymbol{o}}A_2$. This means that at the combined equilibrium configuration, the center of the field and the two centroids must lie on the same straight line where the center of the field is between the two centroids (Figure 5). From now on, let us call this straight line by G.

3.1.2 Pushing Parts away from Each Others

Once the two parts stop at the combined stable equilibrium $\boldsymbol{q}$, the field $\mathcal{J}$ is turned off and an inverse squeeze

[2]commonly assumed that other physical forces such as friction are negligibly small

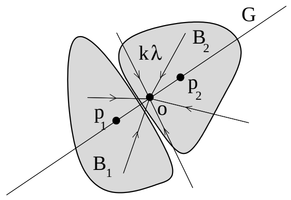


Figure 5: B_1 and B_2 in equilibrium under the field $\mathcal{J}$ with their centroids colinear with the field center $\boldsymbol{o}$.

field is activated to separate the parts into different half planes. In the following, we explain how this inverse squeeze field is constructed.

As mentioned earlier, at the combined equilibrium configuration $\boldsymbol{q} = (\boldsymbol{q}_1, \boldsymbol{q}_2)$, we must have $\boldsymbol{F}_1 + \boldsymbol{F}_2 = k\overrightarrow{\boldsymbol{p}_1\boldsymbol{o}}A_1 + k\overrightarrow{\boldsymbol{p}_2\boldsymbol{o}}A_2 = 0$. That is, $(\boldsymbol{o} - \boldsymbol{p}_1)A_1 + (\boldsymbol{o} - \boldsymbol{p}_2)A_2 = 0$ or equivalently $\boldsymbol{o} = \frac{\boldsymbol{p}_1 A_1 + \boldsymbol{p}_2 A_2}{A_1 + A_2}$. Noticing that the position of the centroid of $B_1(\boldsymbol{q}_1) \cup B_2(\boldsymbol{q}_2)$ can be written as $\frac{\boldsymbol{p}_1 A_1 + \boldsymbol{p}_2 A_2}{A_1 + A_2}$, we therefore have that this centroid must be situated at the field center $\boldsymbol{o}$.

Let L be a horizontal line (i.e., parallel to the ξ-axis of the world frame mentioned in Section 2) that is fixed with the plane of force fields and passing through the field center $\boldsymbol{o}$. This line divides the plane into the top half H_t and the bottom half H_b. By a simple fact that a line passing through the centroid of a region cuts the region into two pieces with equal area and that the line L is defined to pass through the centroid of $B_1(\boldsymbol{q}_1) \cup B_2(\boldsymbol{q}_2)$, we obtain that the area of $B_1(\boldsymbol{q}_1) \cup B_2(\boldsymbol{q}_2)$ in H_t is equal to that in H_b (see Figure 6), or more precisely

$$\begin{aligned} &\mathrm{area}([B_1(\boldsymbol{q}_1) \cup B_2(\boldsymbol{q}_2)] \cap H_t) = \\ &\mathrm{area}([B_1(\boldsymbol{q}_1) \cup B_2(\boldsymbol{q}_2)] \cap H_b). \end{aligned} \tag{1}$$

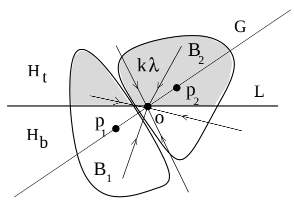


Figure 6: At an equilibrium under $\mathcal{J}$, the areas of the regions occupied by the two parts on both sides of a line passing through the field center are equal (i.e., areas of the shaded and unshaded region are equal).

Because the parts are assumed not to overlap, Equation 1 can be rewritten as

$$\begin{aligned} &\mathrm{area}(B_1(\boldsymbol{q}_1) \cap H_t) + \mathrm{area}(B_2(\boldsymbol{q}_2) \cap H_t) = \\ &\mathrm{area}(B_1(\boldsymbol{q}_1) \cap H_b) + \mathrm{area}(B_2(\boldsymbol{q}_2) \cap H_b). \end{aligned} \tag{2}$$

Let $s_i = \mathrm{area}(B_i(\boldsymbol{q}_i) \cap H_t) - \mathrm{area}(B_i(\boldsymbol{q}_i) \cap H_b), i = 1, 2$. From Equation 2, we have that $s_1 = -s_2$. By the general position assumption, the line G does not coincide with the line L (possibility of the coincidence is virtually zero). The centroids of both parts ($\boldsymbol{p}_1$ and $\boldsymbol{p}_2$) are therefore not on L (Figure 6). This means that, in general, the area of $B_1(\boldsymbol{q}_1)$ in H_t is not equal to that in H_b and, as a result, implies $s_1 \neq 0$. That $s_1 = -s_2$ and $s_1 \neq 0$ suggests that we would be able to push the parts into different half planes if we could have a force field that exerts a force $\begin{pmatrix} 0 \\ s_i \end{pmatrix}$ on B_i (i.e., opposite nonzero force for each part). It turns out that such a force field exists; it is in the form of an inverse squeeze field. Let us call it the field $\mathcal{S}$. Under this field, every point in H_t is under the unit constant field $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ while every point in H_b is under the unit constant field $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ (Figure 7(a)). Because the net force exerted on a part by a unit constant field is a force in the direction of the field with its magnitude equal to the area of the part, it is then easy to see that the force exerted by the field $\mathcal{S}$ on the part $B_i, i = 1, 2$ is $\begin{pmatrix} 0 \\ s_i \end{pmatrix}$ as desired. Note that instead of a simple inverse squeeze field, we may apply two contiguous squeeze fields shown in Figure 7(b) to separate the parts and center them on the line L_t and L_b.

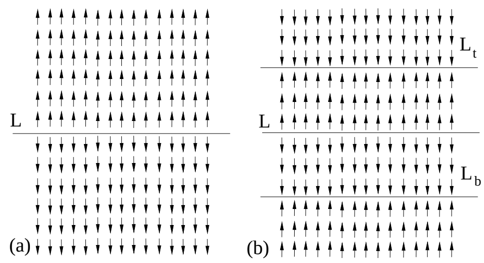


Figure 7: (a) an inverse squeeze field with the axis L, (b) two consecutive squeeze fields, one in the top half plane with the axis L_t and the other in the bottom half with the axis L_b.

Our approach relies on the general position assumption claiming that the event in which $s_1 = 0$ rarely occurs. We might assume uniform distribution of the orientation of the line G to support the claim but this might not be a reasonable assumption for a certain problem under consideration. A strategy that reduces the dependency on the general position assumption should be explored. Also note that the part convexity is assumed to prevent one part to be entangled in concavity of the other during this isolation stage. Therefore, concave parts could, in fact, be handled if entanglement cannot occur.

3.2 Sorting Parts

At this point, one part is in the top half plane H_t and the other part is in the bottom half plane H_b. Still we do not know which part is in which half of the plane. The goal of this stage is therefore to recognize which part is B_1 or B_2. This process is also known as sorting. We propose to perform sorting by applying a sequence of force fields which will move only a specified part to a known region of the plane. This sorting strategy relies on Lemma 3 which is an excerpt from [10]. The lemma specifies a force field that induces a unique stable equilibrium configuration for every part in a given set. The proof of the lemma and the computation of the corresponding stable equilibrium for each part can also be found in the same paper.

Lemma 3 *Consider distinct parts B_i, $i = 1, 2, ..., n$. Let h, k and c be arbitrary positive constants and let d_i be the distance between the centroid of B_i and the pivot point of B_i under the field $\hat{\mathcal{K}} \stackrel{\text{def}}{=} \langle \boldsymbol{o}, h + (k + c)\lambda \rangle$. Any part B_i, $i \in \{1, 2, ..., n\}$ with $d_i > 0$ has a unique stable equilibrium under the combination of the radial field $\hat{\mathcal{J}}(\boldsymbol{o}^*) \stackrel{\text{def}}{=} \langle \boldsymbol{o}^*, h + (2k + c)\lambda \rangle$ and the constant field $\hat{\mathcal{C}} \stackrel{\text{def}}{=} \begin{pmatrix} -kd^* \\ 0 \end{pmatrix}$, where $d^* = \min\{d_i,\ i = 1, 2, ..., n\}$.*

At the beginning of this stage, note that there are two possibilities: (1) B_1 in H_t and B_2 in H_b, and (2) B_1 in H_b and B_2 in H_t. Without loss of generality, let us introduce a sequence of force fields that selectively moves only B_1 to the left half plane.

First, we simultaneously apply

- in the half plane H_t, the combination of the field $\hat{\mathcal{J}}(\boldsymbol{o} + \begin{pmatrix} 0 \\ w \end{pmatrix})$ and the constant field $\hat{\mathcal{C}}$, and
- in the half plane H_b, the combination of the field $\hat{\mathcal{J}}(\boldsymbol{o} - \begin{pmatrix} 0 \\ w \end{pmatrix})$ and the constant field $\hat{\mathcal{C}}$,

where w is a positive constant and the field $\hat{\mathcal{J}}$ and $\hat{\mathcal{C}}$ are defined according to Lemma 3 with $n = 2$. By Lemma 3, under the above force field setup, it is easy to see that when the two parts stop at their stable equilibrium configurations, only two scenarios are possible. They are

1. B_1 in H_t stops at the equilibrium configuration $\boldsymbol{q}_{1,t}$ while B_2 in H_b stops at the equilibrium configuration $\boldsymbol{q}_{2,b}$ (Figure 8(a)), and
2. B_2 in H_t stops at the equilibrium configuration $\boldsymbol{q}_{2,t}$ while B_1 in H_b stops at the equilibrium configuration $\boldsymbol{q}_{1,b}$ (Figure 8(b)),

where $\boldsymbol{q}_{i,t}$ and $\boldsymbol{q}_{i,b}, i = 1, 2$ are the unique stable equilibrium configurations of B_i induced respectively by the combination of the field $\hat{\mathcal{J}}$ and $\hat{\mathcal{C}}$ in the top and bottom half planes. Noticing the position of the center of the field $\hat{\mathcal{J}}$ in each half plane, we have that $\boldsymbol{q}_{1,b} = \boldsymbol{q}_{1,t} - (0, 2w, 0)^T$ and $\boldsymbol{q}_{2,b} = \boldsymbol{q}_{2,t} - (0, 2w, 0)^T$. Also note that the constant w must be set large enough that $B_i(\boldsymbol{q}_{i,t})$ and $B_i(\boldsymbol{q}_{i,b}), i = 1, 2$ do not intersect with the line L.

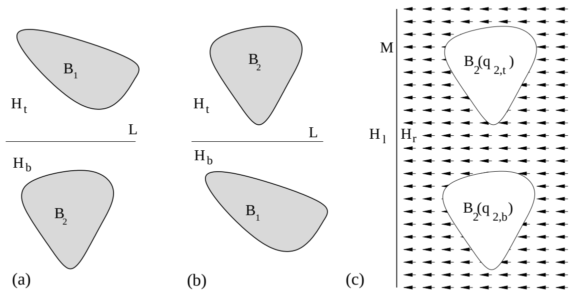


Figure 8: (a)-(b) two possible scenarios (see text), (c) a field that selectively move only B_1 to the left half plane.

Because $\boldsymbol{q}_{i,t}$ and $\boldsymbol{q}_{i,b}, i = 1, 2$ can be computed ([10]), a great deal of uncertainty has been eliminated when the two parts stop. To selectively move only B_1 to the left half plane, we apply the field shown in Figure 8(c). This field is simply a unit constant field $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ applied only in the region $H_r - B_2(\boldsymbol{q}_{2,t}) - B_2(\boldsymbol{q}_{2,b})$ where H_r denotes the right half plane which is bounded on the left by the vertical line M. Regardless of which one of the two scenarios (listed previously) actually happens, the part B_1 always overlap with the constant force field and will move to the left half plane H_l while the part B_2 remains unaffected and stays at either $\boldsymbol{q}_{2,t}$ or $\boldsymbol{q}_{2,b}$. Note that the assumption $A_1 \geq A_2$ given at the beginning ensures that it is impossible that $B_1(\boldsymbol{q}_{1,t}) \subseteq B_2(\boldsymbol{q}_{2,t})$ or $B_1(\boldsymbol{q}_{1,b}) \subseteq B_2(\boldsymbol{q}_{2,b})$ and therefore B_1 must overlap with nonzero force field.

3.3 Positioning Parts

At this point, it is known that the part B_1 is in the left half plane H_l and the part B_2 is in the right half plane H_r. Following [10], we can, again, use Lemma 3 to appropriately set up the field $\hat{\mathcal{J}}$ and $\hat{\mathcal{C}}$ to orient and position each part to a target configuration in its half plane.

4 Discussion and Conclusion

We have introduced a sequence of force fields that brings two distinct convex parts from unknown configurations to target configurations without using any sensory input.

This is a preliminary attempt in using programmable force fields to handle a sensorless multiple object manipulation problem. It clearly demonstrates throughout the paper that action can replace sensor in certain cases. The work is based on an abstract model where control and execution are assumed to be perfect. Of course, tolerance to errors has to be investigated to make the approach useful in a physical implementation. The work raises many issues for further investigation. The most critical questions are: how discretization would affect when continuous force field cannot be realized in a physical implementation?, how long does it take for a part in a given force field to stop at an equilibrium, what is the appropriate damping force model? Besides answering these questions, our future works will focus in studying interaction of multiple parts in other types of force fields. This may give us a strategy for dealing with more parts which is apparently useful for many applications such as part assembly.

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Appendix

Proof of Lemma 1

Let us denote by $\boldsymbol{c} = \begin{pmatrix} c_x \\ c_y \end{pmatrix}$ the position of the pivot point of the part under the field $\mathcal{J}$ in the part's frame (x, y). From the definition, the resultant force induced by the radial field $\mathcal{J}$ vanishes when the pivot point is positioned at the center of the field. More precisely, we must have $k \int\int \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} c_x \\ c_y \end{pmatrix} dxdy = 0$, where all the integrations are performed over the region occupied by the part. Recalling that the area of the part can be written as $\int\int dxdy$, we therefore have $\int\int \begin{pmatrix} x \\ y \end{pmatrix} dxdy = \int\int \begin{pmatrix} c_x \\ c_y \end{pmatrix} dxdy = \begin{pmatrix} c_x \\ c_y \end{pmatrix} A$, where A denotes the area of the part. That is $\boldsymbol{c} = \begin{pmatrix} c_x \\ c_y \end{pmatrix} = \frac{1}{A} \int\int \begin{pmatrix} x \\ y \end{pmatrix} dxdy$. In other words, the pivot point coincides with the centroid of the part. ■

Proof of Lemma 2

Let us denote by P the centroid and by M an arbitrary point of the part. Also, let $\boldsymbol{p}$ and $\boldsymbol{m} = \begin{pmatrix} \xi \\ \eta \end{pmatrix}$ denote the positions of P and M in the world frame when the part is at a configuration $\boldsymbol{q}$. We can write the resultant force induced by the field $\mathcal{J}$ as

$$\begin{aligned} k \int\int (\boldsymbol{o} - \boldsymbol{m}) d\xi d\eta = k \int\int (\boldsymbol{o} - \boldsymbol{p}) d\xi d\eta + \\ k \int\int (\boldsymbol{p} - \boldsymbol{m}) d\xi d\eta, \end{aligned} \tag{3}$$

where all the integrations are performed over the region occupied by the part. It is easy to see that the second term of the right side of Equation 3 vanishes. This is because $k(\boldsymbol{p} - \boldsymbol{m})$ is essentially the force induced by $\mathcal{J}$ at the point M when the part is at the configuration such that the centroid (which is, by Lemma 1, also the pivot point) is positioned at the field's center and the orientation of the part is the same as that of the configuration $\boldsymbol{q}$. Therefore, we obtain the resultant force from only the first term of the right side: $k \int\int (\boldsymbol{o} - \boldsymbol{p}) d\xi d\eta = k\overrightarrow{\boldsymbol{p}\boldsymbol{o}}A$. ■