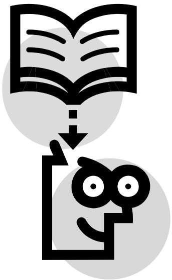
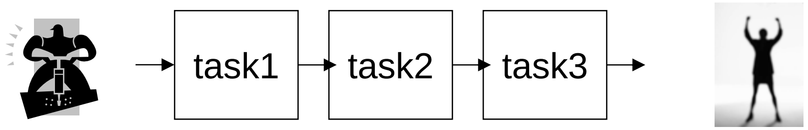
Counting



- <u>Readings:</u>
  Basic counting principles
  Rosen section 4.1-4.3

Basic Counting Principles

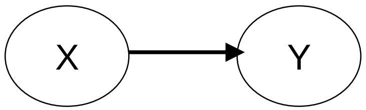
- <u>The product rule</u>
  Suppose a procedure can be broken down into ***a sequence*** of *N* tasks. If there are n_i ways to do the i^{th} task. There are $n_1 n_2 \ldots n_N$ ways to do this procedure.



Basic Counting Principle

<u>Example</u> (Rosen p.303):

How many functions?

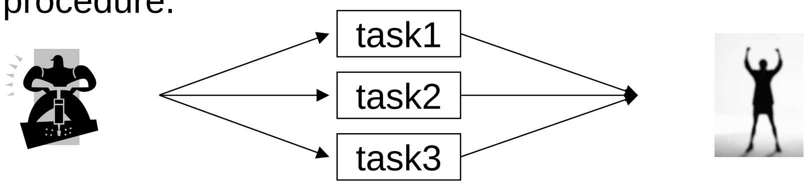


m elements *n* elements

How many one-to-one function?

Basic Counting Principles

- <u>The sum rule</u>
  Suppose a procedure can be divided into ***separate*** *N* tasks which cannot be done at the same time. If there are n_i ways to do the i^{th} task. There are $n_1+n_2+\ldots+n_N$ ways to do this procedure.



Basic Counting Principle

Example (Rosen p.305):

A student can choose a project from one of three lists. The three lists contain 23, 15, and 19 possible projects.

How many possible ways for a student to choose a project?

Example:

A password can contain 6 to 8 characters. Each character can be A-Z. How many possible passwords are there?

Example:

A parking lot consists of a single row of n parking spaces. Only two cars park in this parking lot. How many ways can they park?

How many ways can they park if there can be at most one empty space between them?

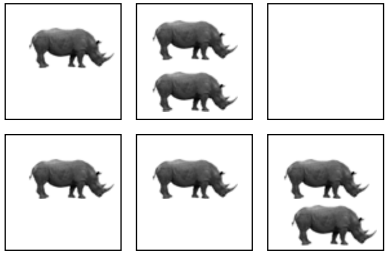
Tree Diagram

Example (Rosen p.309):

How many bit strings of length four do not have two consecutive ones?

The Pigeonhole Principle

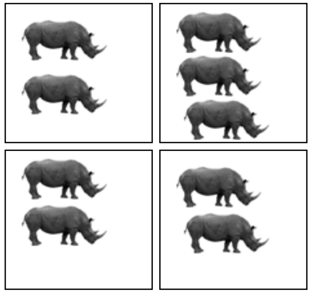
If *k+1* or more objects are placed into *k* boxes, then there are *at least one box containing two or more objects.*



6 boxes
7 objects

The Pigeonhole Principle

If *N* objects are placed into *k* boxes, then there is *at least one box containing at least* $\lceil N/k \rceil$ *objects.*



4 boxes
9 objects
$\lceil 9/4 \rceil = 3$

There is at least one box that contains at least 3 objects.

- <u>Example</u> (Rosen p.315):

  How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?

  How many must be selected to guarantee that at least three hearts are selected?

- <u>Example</u> (Rosen p.315):

  Show that among any *n+1* positive integers not exceeding *2n*, there must be an integer that divides one of the other integers.

e.g.: n=5 {3,4,5,7,8,10}

Permutations

- An <u>ordered</u> arrangement of *r* elements of a set is called an ***r-permutation***
- E.g.: S = {1,2,3}

  1,2 is a 2-permutation of S

  2,1 is another 2-permutation of S

  3,2 is also another 2-permutation of S

  1,2,3 is a permutation of S

  2,1,3 is another permutation of S

Atiwong Suchato
Faculty of Engineering, Chulalongkorn University

Permutations

The number of *r-permutations* of a set with *n* distinct elements is:

$$P(n,r) = n(n-1)(n-2)\ldots(n-r+1)$$

<u>Proof</u>:

Atiwong Suchato
Faculty of Engineering, Chulalongkorn University

- <u>Example</u> (Rosen p.321):

  How many ways are there to select a 1^{st}-prize winner, a 2^{nd}-prize winner, and a 3^{rd}-prize winner from 100 people?

Atiwong Suchato
Faculty of Engineering, Chulalongkorn University

- <u>Example</u>:

  A parking lot consists of a single row of *6* parking spaces. Six people always park here, among them are Aj. Atthasit and Aj. Attawith.

  If Aj. Atthasit can always find a parking spot on the left of Aj. Attawith. How many ways can the six cars park in this parking lot?

Atiwong Suchato
Faculty of Engineering, Chulalongkorn University

Combinations

- An ***r-combination*** of elements of a set is an unordered selection of *r* elements from the set.
- Or a subset, with *r* elements, of the set.

  E.g.: S = {1,2,3,4}

  {1,2,3} is a 3-combination of S

  {3,2,1} is the same as {1,2,3}

Atiwong Suchato
Faculty of Engineering, Chulalongkorn University

Combinations

The number of *r-combinations* of a set with *n* distinct elements is:

$$C(n,r) = n! / r!(n-r)!$$

Proof:

Atiwong Suchato
Faculty of Engineering, Chulalongkorn University

- Example:

  How many ways are there to select a 3 prize winners winner from 100 people (when the three prizes are identical)?

Atiwong Suchato
Faculty of Engineering, Chulalongkorn University

- Example:

  How many bit strings of length *10* contain more than 2 ones?

Atiwong Suchato
Faculty of Engineering, Chulalongkorn University

- Example:

  If only 3 students will get 'A' for this class and they are chosen randomly. What is the probability that a certain student will get the 'A'?

Combinatorial Proof

- A combinatorial proof is a proof that uses counting arguments to prove a theorem rather than some other method such as algebraic techniques.

- Example (Rosen p.323):

  Prove that when *n* and *r* are nonnegative integers with $n \leq r$, $C(n,r) = C(n,n-r)$

- Example :

  Prove that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$