Graphs & Trees

Topics in Graphs

- 9.1 Graphs and graph models
- 9.2 Graph Terminology
- 9.3 Representing graph / Isomorphism
- 9.4 Connectivity
- 9.5 Euler and Hamilton paths
- 9.7 Planar graph
- 9.8 Graph coloring

Topics in Trees

- 10.1 Intro to trees
- 10.2 Binary search trees
- 10.3 Tree traversal
- 10.4 Spanning trees

Today's Topics

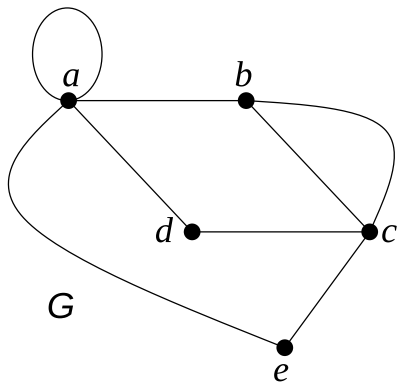
- Graph Definition
- Terminology
- Simple Graphs, Multigraphs, Pseudographs
- Directed Graphs
- Degrees
- Special Types of Graph
- Bipartite Graphs
- Subgraph
- Union of Graphs

Graphs

$G = (V,E)$

$V = \{v_1, v_2, \ldots, v_n\}$ ← Set of Vertices

$E = \{e_1, e_2, \ldots, e_k\}$ ← Set of Edges

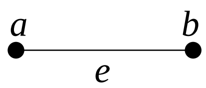


$V = \{a,b,c,d,e\}$

$E = \{\{a,a\},\{a,b\},\{a,d\}, \{a,e\},\{b,c\},\{b,c\}, \{c,e\},\{d,c\}\}$

G has 5 vertices and 8 edges.

Terminology



e is **incident** with a and b.
a is an **end point** of e.
b is the other end point of e.
a is **adjacent** to b.
b is also adjacent to a.

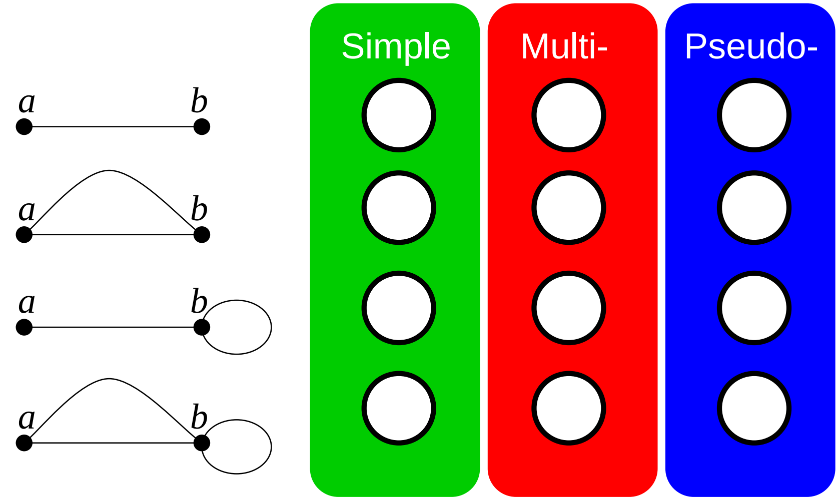
Simple Graph, Multigraph, Pseudograph

Simple Graph
No more than 1 edge between any pair of vertices. No loops.

Multigraph
There can be more than 1 edge between any pair of vertices. No loops.

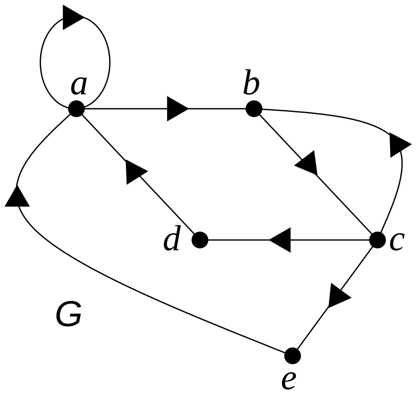
Pseudograph
Any graph.

Simple Graph, Multigraph, Pseudograph



Directed Graphs

- Edges are described using “ordered pairs”.



$E = \{(a,a),(a,b),(d,a),(e,a),(b,c),(c,b),(c,e),(c,d)\}$

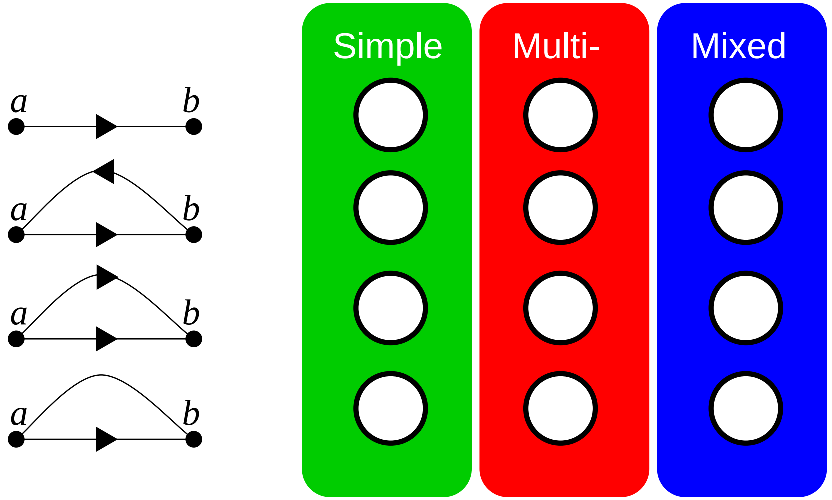
Directed Graphs

Simple Directed Graph
No repeated edges (order matters). No loops

Directed Multigraph
There can be repeated edges and/or loops.

Mixed Graph
There are both directed and undirected edges.

Directed Graphs



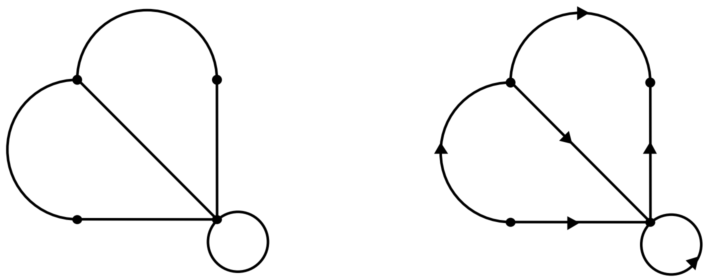
Degree of a Vertex

- Degree of v, $\deg(v)$ = number of edges incident with v
- Out-degree of v, $\deg^+(v)$ = number of edges, each of which has v as their initial vertex.
- In-degree of v, $\deg^-(v)$ = number of edges, each of which has v as their end vertex.

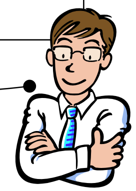
A loop contributes **twice** to the degree of a vertex.

A directed loop contributes **once** to the in-degree and **once** to the out-degree.

Degree of a Vertex



How many edges are there in a graph with 10 vertices each of degree 6?



The Handshaking Theorem

Let $G = (V,E)$ be an undirected graph with e edges, then

$$2e = \sum_{v \in V} \deg(v)$$

The Number of Vertices with Odd Degrees

In an undirected graph, there must be an even number of vertices with odd degree.

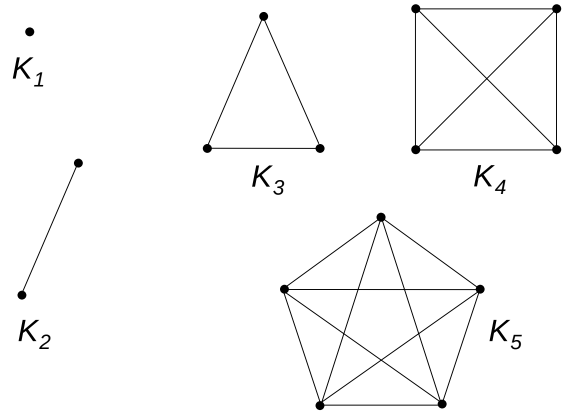
Proof:

In-degree = Out-degree

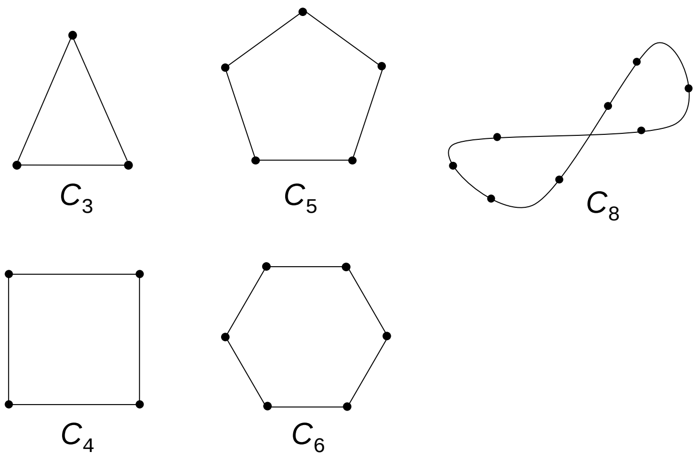
Let $G = (V,E)$ be a directed graph, then

$$\sum_{v \in V} \deg^{-}(v) = \sum_{v \in V} \deg^{+}(v) = |E|$$

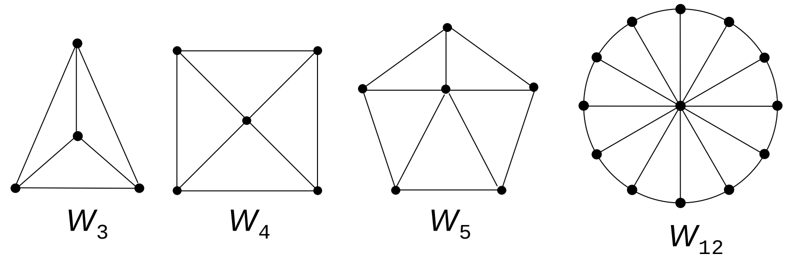
Complete Graphs



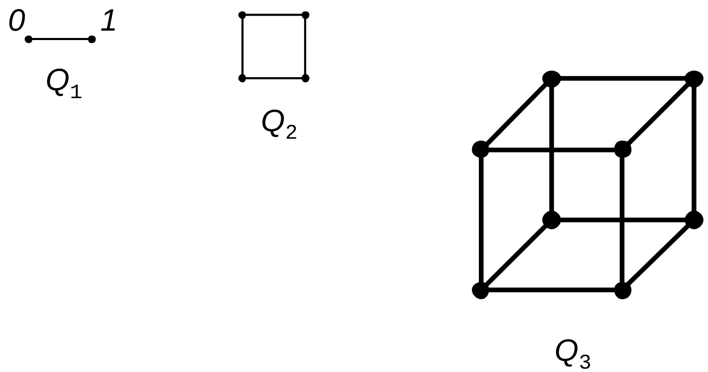
Cycles



Wheels



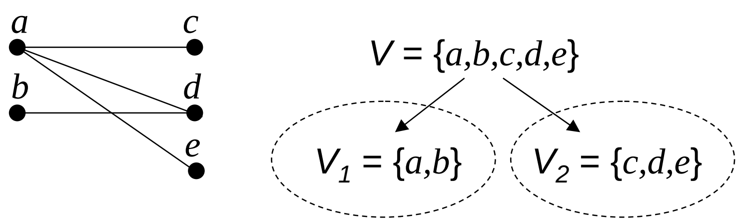
n–Dimensional Hypercube

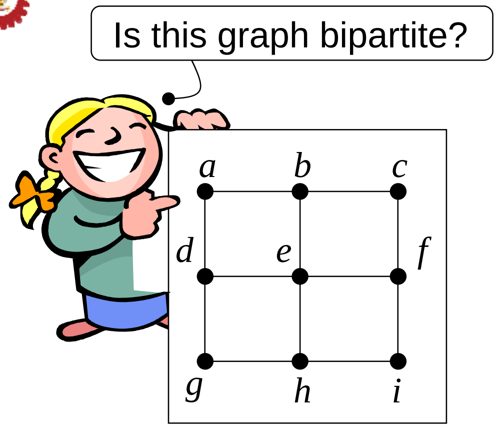


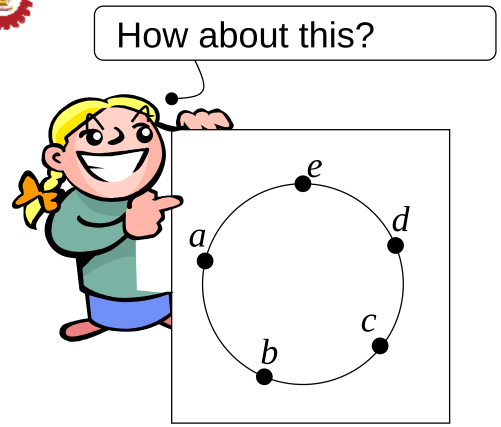
Bipartite Graphs

A simple graph $G = (V,E)$ is called **bipartite** ↔

V can be partitioned into V_1 and V_2 so that no edges connects vertices in the same partition.







Complete Bipartite Graphs

- Vertices are partitioned into a set of *m* vertices and a set of *n* vertices.
- There is an edge between two vertices ↔ one vertex is in the first subset and the other vertex is in the second subset.

Complete Bipartite Graph, $K_{m,n}$

Complete Bipartite Graphs

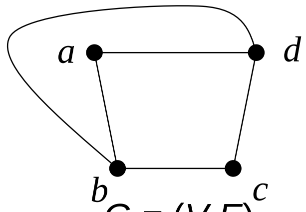
$K_{3,3}$ $K_{4,2}$

Subgraphs

A **subgraph** of $G=(V,E)$ is a graph $H=(W,F)$ where $W \subseteq V$ and $F \subseteq E$

A subgraph of H of G is a **proper subgraph** of G if $H \neq G$

Subgraphs



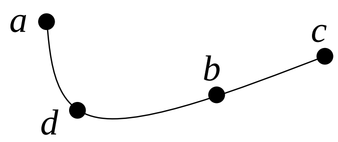
$G = (V,E)$

$V = \{a,b,c,d\}$

$E = \{\{a,b\},\{b,c\},\{c,d\},\{a,d\},\{b,d\}\}$

$W = \{a,b,c,d\}$

$F = \{\{b,c\},\{a,d\},\{b,d\}\}$

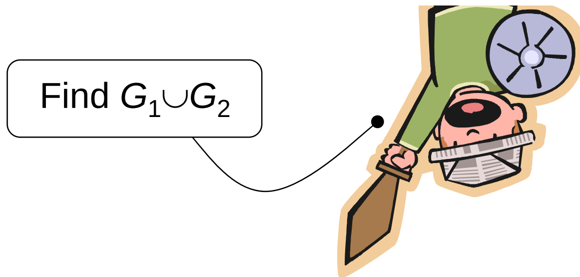


$H = (W,F)$

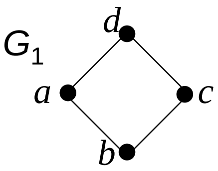
Union

The **union** of two simple graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is the simple graph $H=(W,F)$ where $W = V_1 \cup V_2$ and $F = E_1 \cup E_2$

Denoted by $G_1 \cup G_2$



G_1



G_2