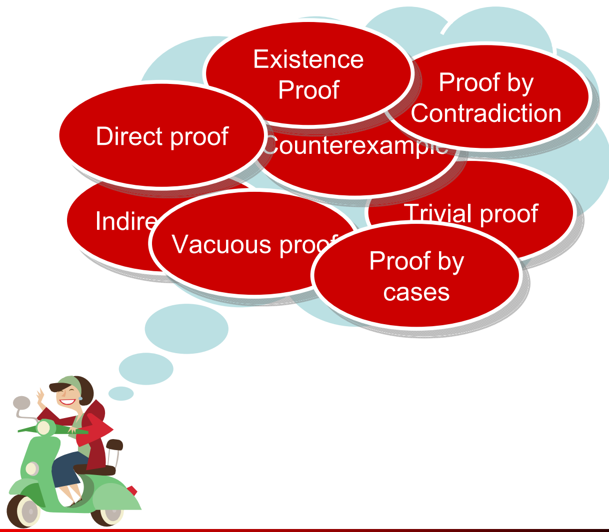
Methods of Proving Theorems



Understanding How theorems are stated

- Many theorems concerns elements in a domain, such as integers or real numbers.

If x > y, where x and y are *positive real numbers*, then $x^2 > y^2$

really means

For all positive real numbers, if x > y, then $x^2 > y^2$

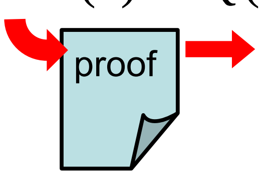
Universal instantiation / Universal generalization

"If *n* is an even integer, n^2 is an even integer"

$$\forall_{n \in Z_{even}} (P(n) \rightarrow Q(n))$$

$P(n)$: n is an even integer.
$Q(n)$: n^2 is an even integer.

$\therefore P(c) \rightarrow Q(c)$ Universal Instantiation



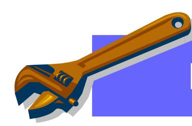
$P(c) \rightarrow Q(c) \equiv \mathrm{T}$

When c is any even integer

$$\forall_{n \in Z_{even}} (P(n) \rightarrow Q(n)) \equiv \mathrm{T}$$

Universal Generalization

Proving p→q



- Direct Proof
- Proof by Contraposition
- Vacuous Proof
- Trivial Proof
- Proof by Contradiction

Proving p→q

- Direct Proof
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Direct Proof: Show that if **p** is true, **q** must be true.

Proof by Contraposition: $p \rightarrow q \equiv \neg q \rightarrow \neg p$

Show that if **¬q** is true, **¬p** must be true.

- Example (Rosen Ex.1 p.77):
  Show that "If *n* is an odd integer, n^2 is an odd integer"

- Example (Rosen Ex.1 p.80):
  Show that "If *n* is an integer and n^2 is odd, then *n* is odd."

Example (Rosen Ex.4 p.78):

Prove that if $n = ab$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

The real number r is rational if there exist integers p and q with $q \neq 0$ such that $r = p/q$

Example (Rosen Ex.6 p.79):

Prove that the sum of two rational numbers is rational.

Proving $p \rightarrow q$

- Direct Proof
- Proof by Contraposition
- **Vacuous Proof** — Show that **p** is false. So, **$p \rightarrow q$** is always true.
- **Trivial Proof** — Show that **q** is true. So, **$p \rightarrow q$** is always true.
- Proof by Contradiction

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Proving $p \rightarrow q$

- Example (Rosen Ex.5 p.78)

  $P(n)$ = "If $n > 1$, then $n^2 > n$"

  Show that $P(0)$ is true.

- Example (Rosen Ex.6 p.79)

  $P(n)$ = "If a and b are positive integers with $a \geq b$, then $a^n \geq b^n$"

  Show that $P(0)$ is true.

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