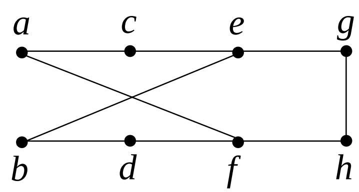
Graphs & Trees (Lecture 3)

Today's Topics

- Euler Paths & Circuits
- Hamilton Paths & Circuits

Euler Paths

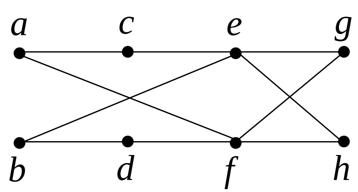
An **Euler path** in a graph is a *simple* path containing *every edges* of that graph.



An Euler path =

Euler Circuits

An **Euler circuit** in a graph is a *simple* circuit containing *every edges* of that graph.



An Euler circuit =

Conditions for Euler Circuits

A connected multigraph with at least two vertices has an Euler circuit ↔ each of its vertices has even degree.

Proof:

Necessary condition

G has an Euler Circuit → each of its vertices must have even degree.

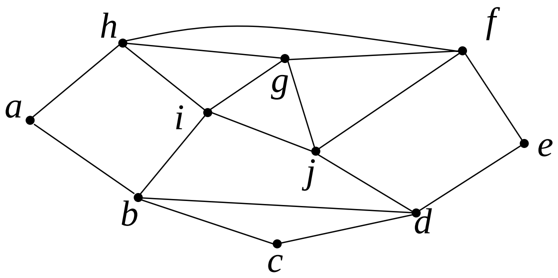
Sufficient condition

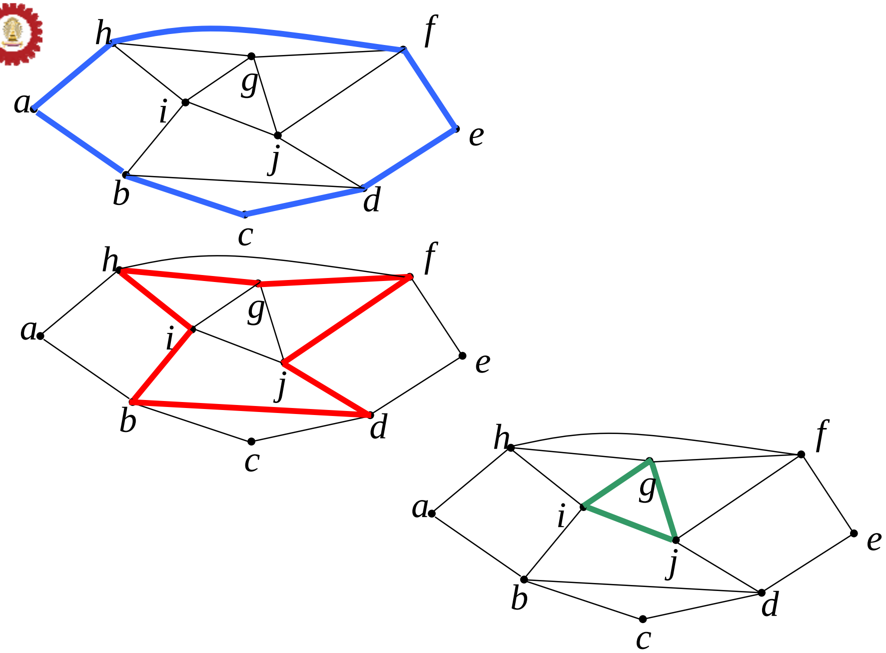
Each of the vertices in *G* has even degree → *G* has an Euler Circuit.

G has an Euler Circuit $\rightarrow$ each of its vertices must have even degree.

Each of the vertices in G has even degree $\rightarrow$ G has an Euler Circuit.

Finding an Euler Circuit

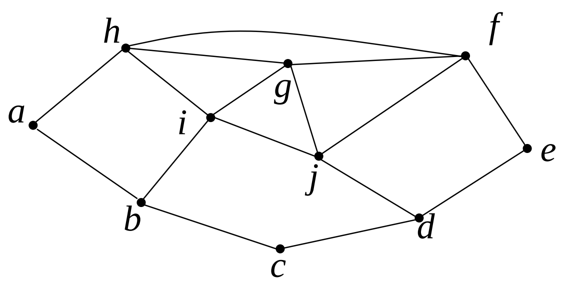


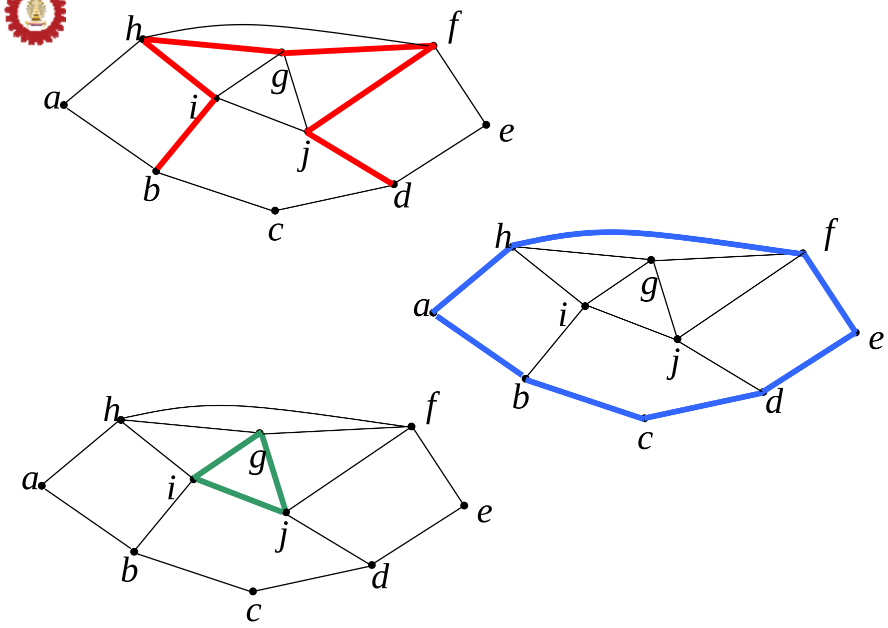


Conditions for Euler Paths

A connected multigraph with at least two vertices has an Euler path ↔ it has exactly 2 vertices with odd degree.

Finding an Euler Path

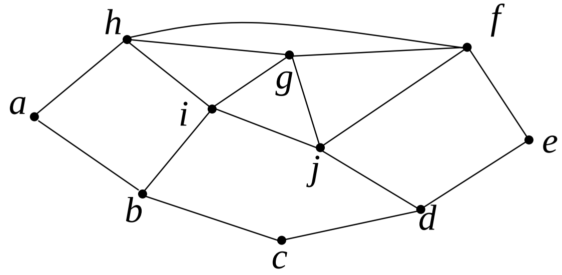




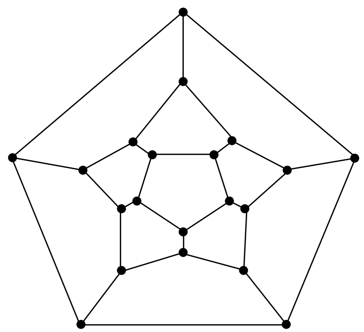
Hamilton Paths and Circuits

A **Hamilton path** in a graph is a *simple* path that passes through *every vertex* of the graph *exactly once*.

For $G=(V,E)$ and $V = \{v_1, v_2, \ldots, v_n\}$, the simple circuit $v_1,v_2,\ldots,v_n,v_0$ is a **Hamilton circuit** if $v_1,v_2,\ldots,v_n$ is a Hamilton path.



Iconian Puzzle



Conditions for Hamilton Circuits

- No ‘necessary & sufficient’ conditions exist.
- Certain properties can be used to show that no Hamilton circuits exist. E.g. degree one vertex.
- Both edges incident of a vertex of degree two must be part of any Hamilton circuit.
- While constructing a Hamilton circuit, if a vertex has already passes through, all remaining edges of that vertex can be removed from consideration.

Some Sufficient Conditions

If G is a simple graph with n vertices ($n \geq 3$) such that the degree of every vertex in G is at least $n/2$, then G has a Hamilton circuit.

If G is a simple graph with n vertices ($n \geq 3$) such that $\deg(u)+\deg(v) \geq n$ for every pair of non-adjacent vertices u and v in G, then G has a Hamilton circuit.