# Graphs & Trees (Lecture 4)

## Today's Topics

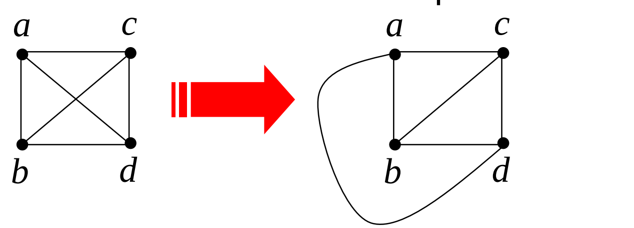
- Planar Graphs
- Graph Coloring



# Planar Graph

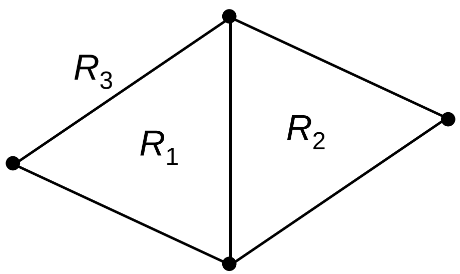
A graph is planar if it can be drawn in the plane without any edges crossing.

Planar Representation



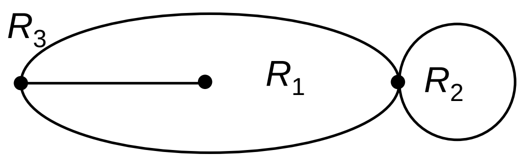



# Regions




Degree of Region

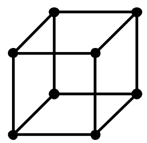
$\deg(R_1) = 3$
$\deg(R_2) = 3$
$\deg(R_3) = 4$



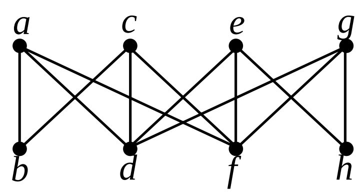

$\deg(R_1) = 4$
$\deg(R_2) = 1$
$\deg(R_3) = 3$

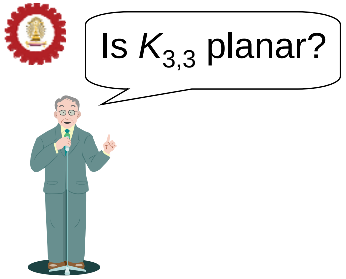


# Planar or Not?

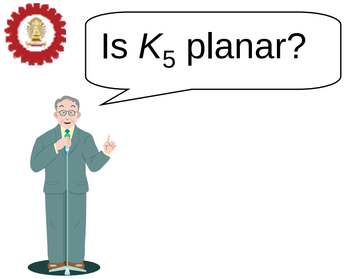


$Q_3$










# Euler's Formula

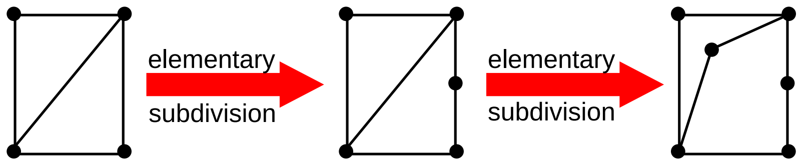
Let $G$ be a connected planar simple graph with
$e$ = Number of edges
$v$ = Number of vertices
$r$ = Number of regions in a planar representation of $G$.

Then,

$$\boxed{r = e - v + 2}$$

# Elementary Subdivision

- An **Elementary Subdivision** is an operation that removes an edge $\{u,v\}$ and adding a new vertex w together with edges $\{u,w\}$ and $\{w,v\}$

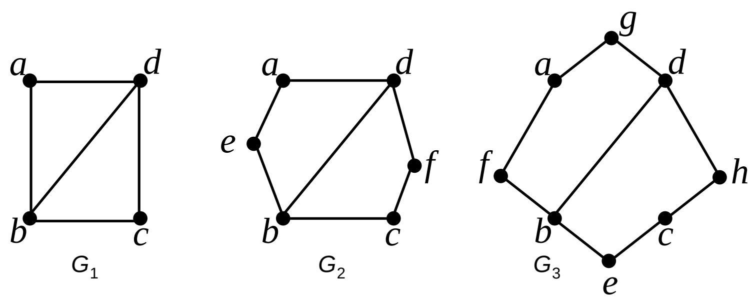





# Homeomorphism

- $G=(V,E)$ and $H=(W,F)$ are **homeomorphic** if they can be obtained from the same graph by a sequence of elementary subdivisions.

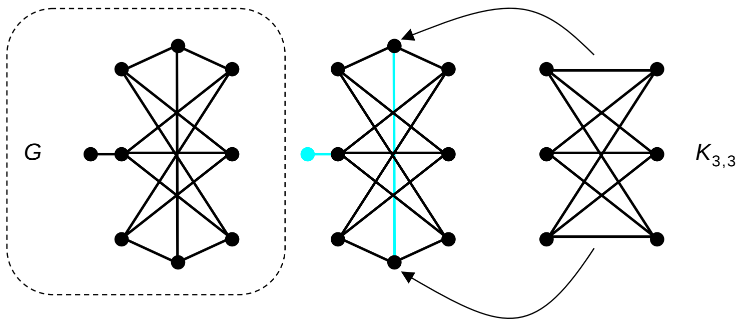


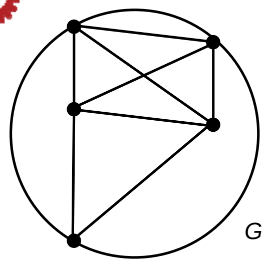





# Kuratowski's Theorem

A graph is nonplanar ↔ it contains a subgraph homeomorphic to $K_{3,3}$ or $K_5$.






Use Kuratowski's theorem to show that *G* is nonplanar.



# Graph Coloring

- A **coloring** of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.
- The **Chromatic Number** of *G*, $\chi(G)$, is the least number of colors needed for a coloring.



Find the chromatic number of:

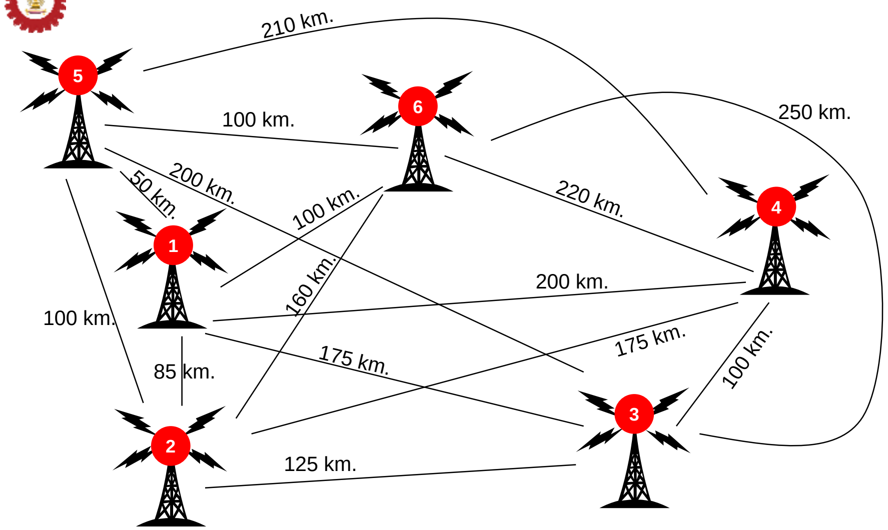
$C_n$

$W_n$

$K_{m,n}$

$Q_n$

$K_n$




Two radio stations cannot use the same frequency channel when they are within 150 km. of each other. How many channels are needed?

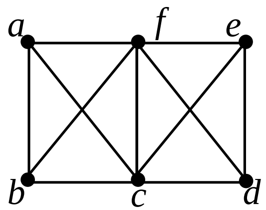




# The Four Color Theorem

The chromatic number of a planar graph is no greater than 4.

Find the chromatic number of *G*.




*G*