Adder Circuit on IBM Universal Quantum Computers

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***Abstract*— This paper reports the experiments on IBM quantum devices. We design an adder circuit based on Thomas G. Draper proposal. This circuit is a part of a general circuit to build Shor prime number factorization algorithm. We perform the experiments on three IBM quantum devices. The result of the experiment shows that on IBM Essex device, the best possible adder for one qubits has 100% accuracy. However, the results for two qubits adder circuit are much worse. The details analysis of the accuracy of three IBM devices are reported.**

Keywords*— *quantum adder circuit, quantum computer, IBM quantum computer

I. Introduction

In the past decade quantum algorithms has been proposed that including the famous algorithm like Grover search algorithm [1] in 1996 and Shor prime number factorization algorithm [2] in 1997. In recent year many companies are pushing for building quantum computers. They also open their quantum computers for researchers around the world to access them. IBM [3] and Rigetti Computing [4] are the two frontier companies which open their systems for the researcher. This is good opportunity for gaining experience on real quantum devices. There has been an interest in implementing algorithms on quantum computers. In 2003 Stephan Beauregard [5] proposed a generalize circuit for Shor's algorithm which is interesting because it is quite simple and it uses only 2n+3 qubits to build the circuit. This proposed circuit is based on the quantum adder circuit by Thomas G. Draper [6] in 1998.

In this work, we perform the experiment on an adder circuit which is a part of Beauregard design on the IBM quantum computer. The results from running the circuit on the actual quantum computer can be very different from running it on the software simulation. This is due to the fact that building quantum computers is still in an early stage. Hence it is interesting to understand this behavior. Hopefully, this report should help other researchers who want to try their programs on real quantum computers to obtain better results.

II. Background

A. Quantum Fourier Transform

The quantum Fourier transform is the one of the three classes that categorized by Peter W. Shor in "Why Haven't More Quantum Algorithms Been Found?"[7]. It is an algorithm for a quantum computer that will run faster than any algorithm on classical computers. He also uses the behavior of quantum Fourier transform in his famous Shor's algorithm for period finding of $f(x) = a^r \bmod N$.

$$|j > \rightarrow \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{\frac{2\pi ijk}{N}} |k > \quad (1)$$

The quantum Fourier transform represents as equation (1) when |k> are orthonormal basis vector |0>, …, |N-1> transformed to |j> state vector. It can be rewritten as equation (2). The Hadamard (H) and controlled rotation (R_k) gates denotes the unitary transformation written down as equation (3) and (4). The Figure 1 shows the circuit for quantum Fourier transform for demonstrating 3 qubits. It represents equation (2) with the H and R_k gates.

$$|j_1 1, \ldots, j_n > \rightarrow \frac{(|0>+e^{2\pi i0.j_n}|1>)\ldots(|0>+e^{2\pi i0.j_1 j_2 \ldots j_n}|1>)}{2^{n/2}} \quad (2)$$

$$H \equiv \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (3)$$

$$R_k \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{bmatrix} \quad (4)$$

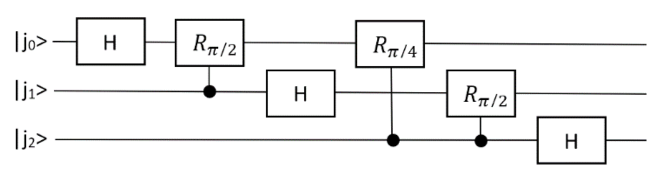

Fig. 1. Example of quantum Fourier transform 3 qubits circuit

B. Adder Circuit

The quantum addition was introduced by Thomas G. Dapper [6] in 1998. This quantum adder circuit use an idea of quantum Fourier transform (5) and the Fig 2 shows an example of adder circuit with 2 qubits based on that report.

$$|\emptyset_{n-1}(a) > \rightarrow \frac{1}{\sqrt{2}}(|0 > +e(0.a_{n-1} \ldots a_0 + 0.b_{n-1} \ldots b_0)|1 >) \quad (5)$$

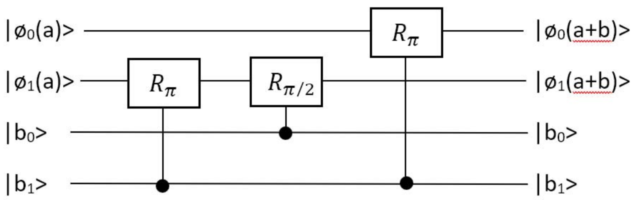

Fig. 2. Example of quantum addition 2 qubits circuit

C. *Circuit on IBM Q Experience*

IBM Q Experience is a public cloud platform for everyone to construct a quantum circuit and run it on a simulator or IBM superconducting quantum devices. The company opens multiple ways to construct and run a circuit including drag and drop user interface, quantum assembly program QASM) [8] and a Python library named 'Qiskit' [9].

The results from IBM devices are a probability of each possible solutions. The circuit must be run multiple times to observe the possible answer. Fig 3 shows a result of our two qubits adder circuit performed the operation between two binary numbers '11' and '10' on IBM Essex device. The best possible answer is '001' with probability at 27.98%, the second answer is '101' with probability at 24.10% and the third answer is '100' with probability at 9.75%.

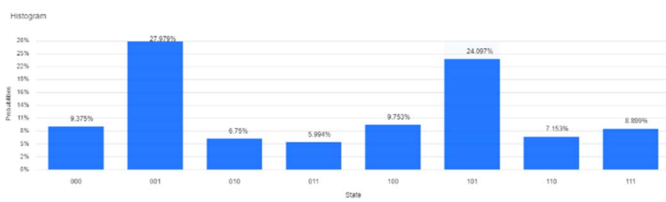

Fig. 3. Addition result from Essex with input 11 and 10.

III. QUANTUM ADDITION EXPERIMENTS

The adder circuit in our experiment is shown in Fig 4. It has five stages. The first stage is the initialization of inputs. The second stage is to prepare registers to be in a quantum Fourier transform state. We use Hadamard gate and controlled phase rotation gate. The third stage is the quantum addition using the controlled phase rotation gate. The fourth and fifth stages are the measurement of the results on registers by inverse the quantum Fourier state and perform a measurement of the classic bits. One important part of our experiments are the adder circuits constructed without any prior knowledge of the qubit's connectivity or gate error on quantum devices.

The experiments are conducted on three different types of IBM quantum universal computer and one IBM quantum simulator. The results from the simulator are used as the reference. Three IBM quantum computers are different in the number of qubits and qubits connectivity topology. There are two IBM quantum devices with 5 qubits (ibmq_5_yorktown and ibmq_essex) and one 15 qubits (ibmq_16_melbourne). The two 5 qubits quantum devices have different qubits connectivity as shown in Fig 5 and Fig 6. The one 16 qubits connectivity topology is shown in Fig 7. We perform addition operations with two numbers that fit in quantum devices including one and two bits.

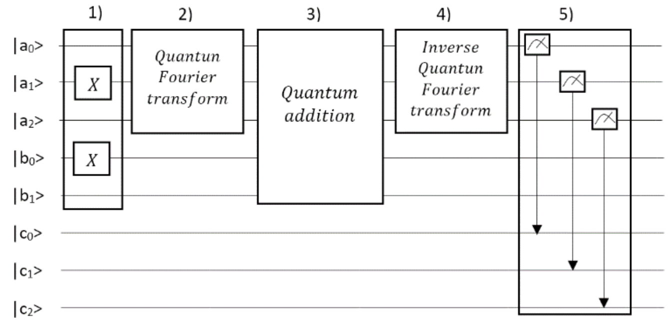

Fig. 4. Experiment adder circuit with state a is 10 and b is 01.

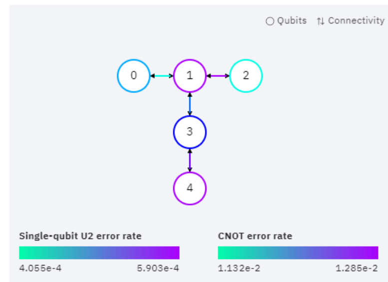

Fig. 5. qubit's connectivity and error rate on IBM 5 qubits Essex [3]

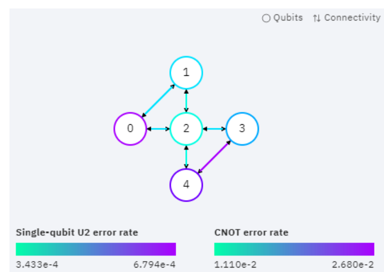

Fig. 6. qubit's connectivity and error rate on IBM 5 qubits Yorktown [3]

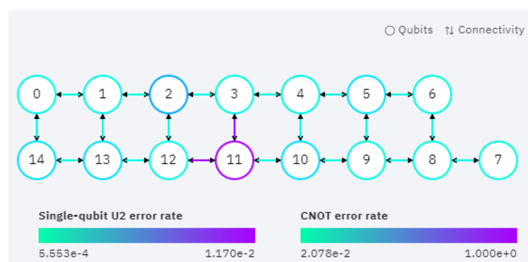

Fig. 7. qubit's connectivity and error rate on IBM 16 qubits Melbourne [3]

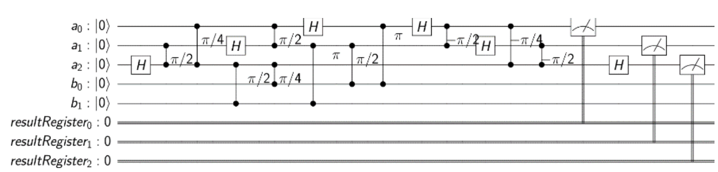

Fig. 8. The modified 2 qubits adder circuit.

For the two qubits quantum addition circuit to be able to run on IBM five qubits devices, it requires a slight modification from Thomas G Dapper [6] proposal because of the limitation of devices that have only five qubits. The new adder circuit for two bits input are designed by using two qubits for each input number and one carry qubits. The experimental adder circuit is modified from Thomas G Dapper three qubits adder by removing one control rotation gate between two most significant qubits followed by removing the most significant qubit on second input register (register $|b_n>$ in Fig 8).

IV. RESULTS

The experimental results are shown in Table 1. The table shows the accuracy from three quantum adder circuits that are designed for one qubit and two qubits number from three types of IBM quantum devices. In the experiments, we run the circuits in the test 8,192 times and report a maximum as a solution. The one qubit addition results on IBM Essex device are promising because it adds all two numbers correctly. This result is better than the result from other IBM Yorktown and Melbourne that achieve only 50% correct. However, on the two qubits addition the accuracy rate is lower. There is no correct answer from IBM Melbourne. The best device is IBM Essex with 25.00% of accuracy followed by IBM Yorktown with 6.25%.

TABLE I. IBM QUANTUM ADDER CIRCUIT ACCURACY RATE

Device	accuracy rate (in percentage)	
	adder 1 qubits	***adder 2 qubits***
IBM Essex 5 qubits	100.0%	25.0%
IBM Yorktown 5 qubits	50.0%	6.25%
IBM Melbourne 15 qubits	50.0%	0.0%

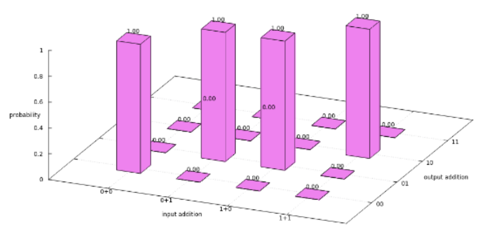

Fig. 9. The result of one qubit experiment on IBM simulator.

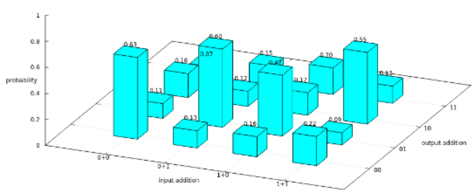

Fig. 10. The result of one qubit experiment on IBM Essex.

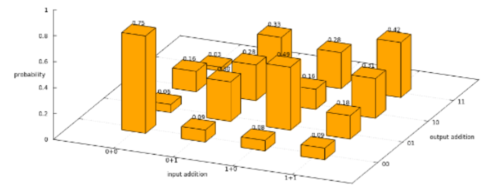

Fig. 11. The result of one qubit experiment on IBM Yorktown.

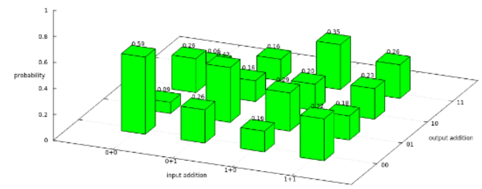

Fig. 12. The result of one qubit experiment on IBM Melbourne.

Another perspective of measurement is demonstrated in Fig 9, 10, 11 and 12 for one qubit addition experiment and Fig 13, 14, 15 and 16 for two qubits addition experiment. The xy plane is a ground plane, x axis is inputs in binary, y axis is outputs in binary and z axis is the probability of output normalized from zero to one. The color of each figure denotes different devices. The violet color is the IBM simulator, the cyan color is the IBM Essex, the orange is the IBM Yorktown and the green is the IBM Melbourne.

The one qubit addition in Fig 9 (IBM simulator) is the ideal case. It shows high confidence on a single correct output. When comparing the Fig 9 with Fig 10, 11 and 12, the most similar figure is the Fig 10 which is the result on IBM Essex. All peaks on Fig 10 are at the same position on Fig 9. The Fig 11 (IBM Yorktown) has two peaks at difference positions from Fig 9 but both second peaks on Fig 11 are at peak positions of Fig 9. This means the correct outputs are second peaks (on input '0+1' and '1+1'). This also happens on Fig 12 (IBM Melbourne) compared to Fig 9. It has one second peak (on input '1+0') at the correct position and one third peak (on input '1+1') at the correct position.

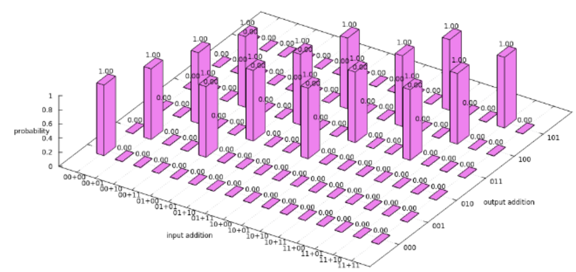

Fig. 13. The result of two qubits experiment on IBM simulator.

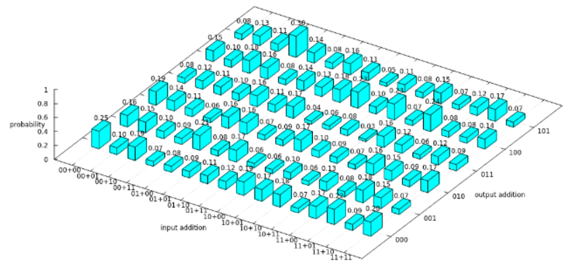

Fig. 14. The result of two qubits experiment on IBM Essex.

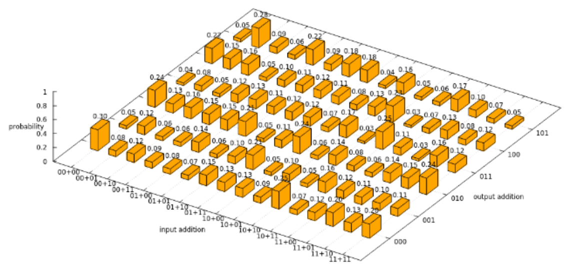

Fig. 15. The result of two qubits experiment on IBM Yorktown.

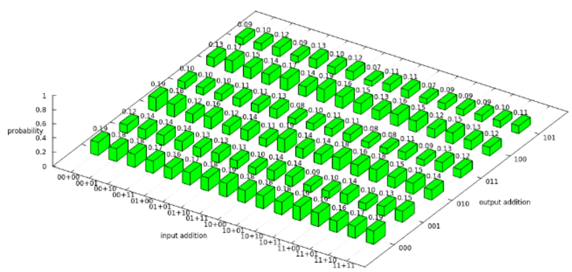

Fig. 16. The result of two qubits experiment on IBM Melbourne.

The result of two qubits experiment on Fig 14, 15 and 16 have no distinct peaks visually. Especially on Fig 16 (IBM Melbourne), all probabilities are almost flat. The resulting peak probability at output ‘000’. The Fig 14 (IBM Essex) and 15 (IBM Yorktown) have some peaks. The input ‘00+00’ have the same peak position at output ‘000’. It also matched with Fig 13 (IBM Simulator).

V. CONCLUSIONS

This work presents the experimental results of adder circuit on three different IBM quantum computers. The circuit is constructed based on the proposal of Thomas G. Dapper [6]. The placement of the circuit onto the real devices is done without any prior knowledge of qubit’s connectivity on IBM devices. This is a realistic situation when we use IBM devices as we do not know in advance how the circuit generation tools produce the layout. The one qubit addition on the Essex device has the best accuracy. It has completed all addition tests without any incorrect output. This is compared to Yorktown or Melbourne which only has 50% accuracy. However, on the two qubits addition the accuracy of all devices is substantially dropped. The Essex has accuracy only 25% on two qubits and the Melbourne has accuracy 0%. We also take a deep look into the accuracy using the maximum probability as the result from addition. It is possible that each possible result is not significantly different. For the Essex device, when the result is correct, the difference tends to go lower when increasing the number of qubits. The Melbourne device for two qubits addition has accuracy 0% but the difference between the maximum probability and the correct answer probability is only 4%. This fact suggests that there should be a better way than using the maximum probability as the only answer from the addition circuit. Based on these results we might not be able to construct a larger addition circuit without any knowledge of quantum device architecture. We hope that our discovery will allow other researchers to improve the quality of their results in running circuits on IBM quantum devices.

ACKNOWLEDGMENT

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