A Tutorial on Evolutionary Multiobjective Optimization

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Overview

❶ **The Big Picture:**

Optimization and evolutionary computation

❷ **The Construction Kit:**

Design issues and algorithmic concepts

❸ **The Pieces Put Together:**

Example of an algorithm variant

❹ **The Big Question:**

Performance of evolutionary algorithms

❺ **The Challenge:**

Standard interface for search algorithms

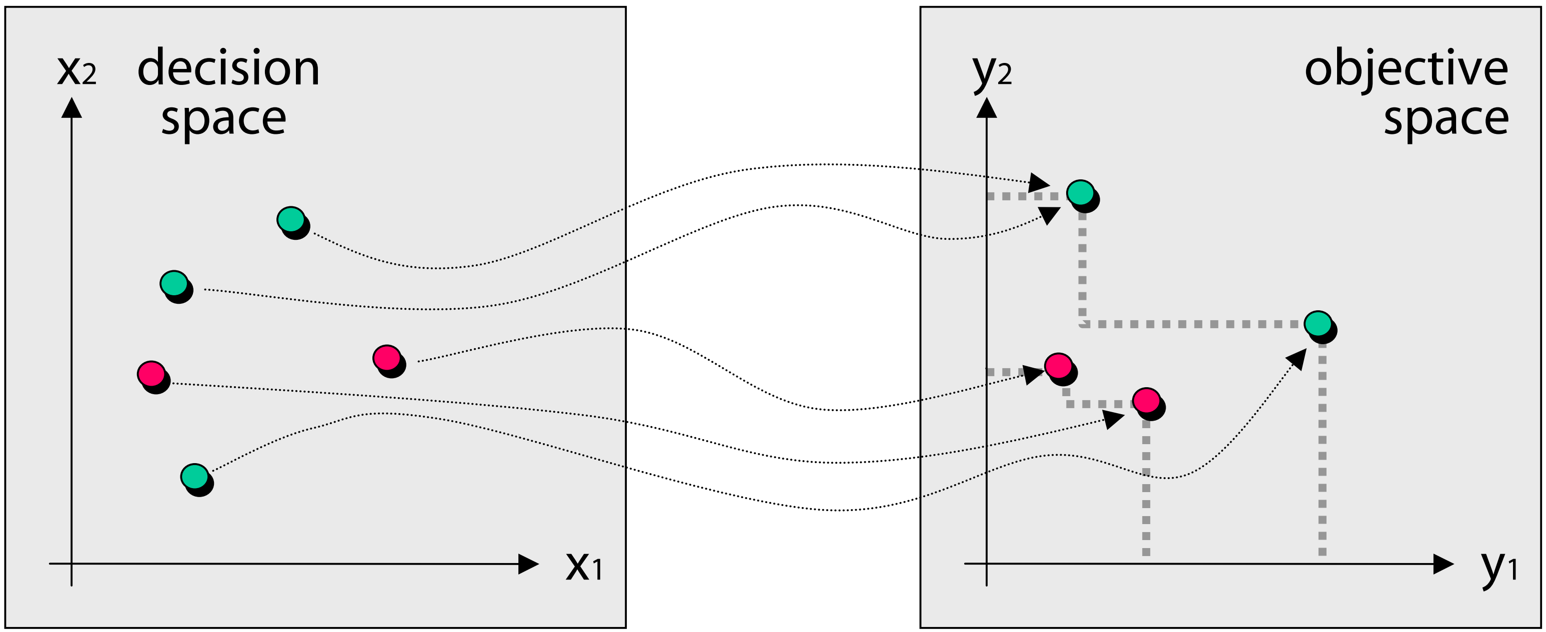
❻ **The End:**

Conclusions and outlook

Decision and Objective Space

Pareto set — Pareto front

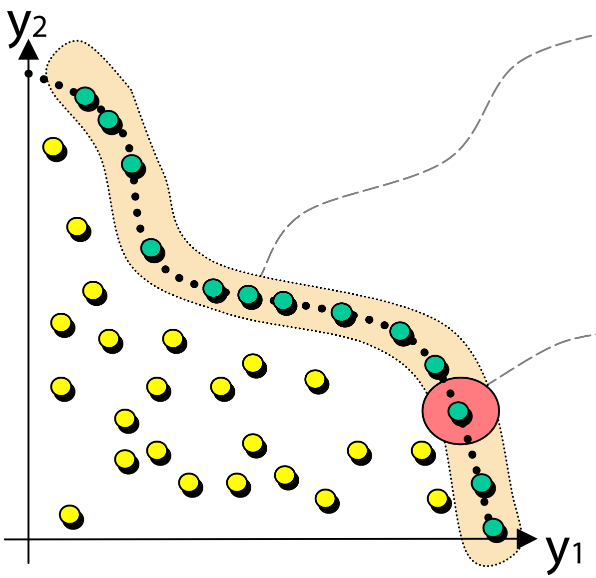
Pareto set approximation — Pareto front approximation



$(x_1, x_2, \ldots, x_n) \longrightarrow \mathbf{f} \longrightarrow (y_1, y_2, \ldots, y_k)$

search ⟵ **evaluation**

Optimization and Decision Making



Pareto optimality:
defines set of optimal trade-offs
(all objectives equally important)

Decision making:
choose best compromise
(based on preference information)

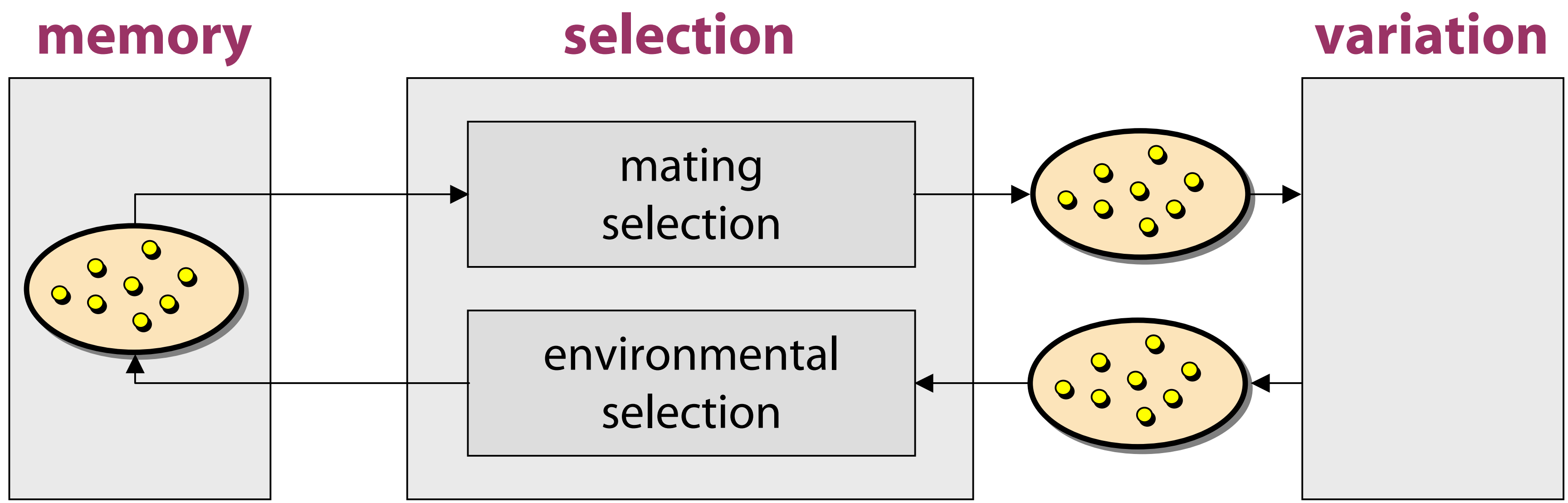
❶ Decision making before search (define single objective)

❷ Decision making after search (find/approximate Pareto set first)

❸ Decision making during search (guide search interactively)

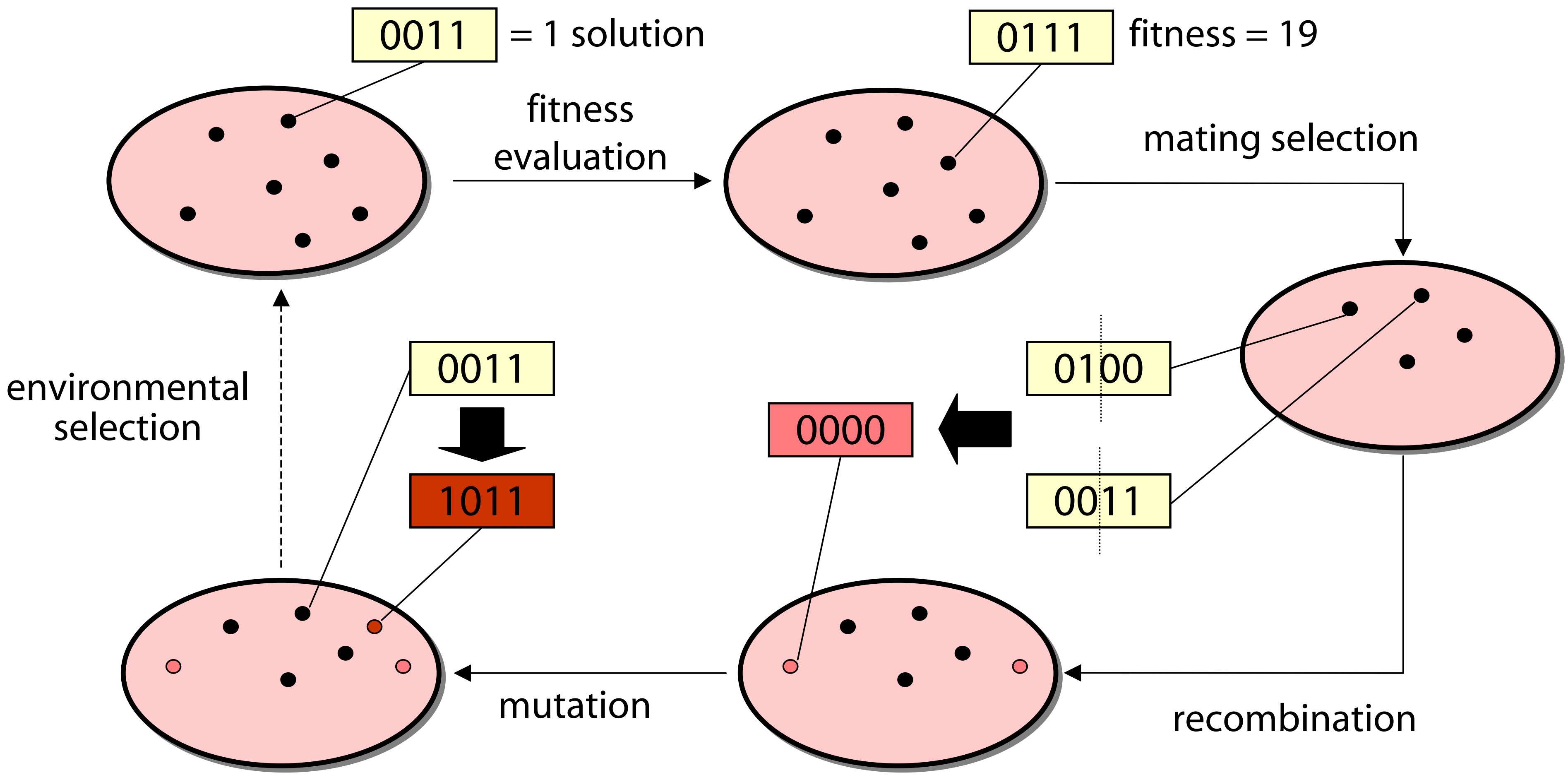
❹ Combinations of the above

Stochastic Search Algorithms



EA	≥ 1	both	≥ 1 ≥ 1	N : M randomized
TS	1	no mating selection	1 ≥ 1	1 : M deterministic
SA	1	no mating selection	1 ≥ 1	1 : M randomized
ACO	1	neither	1 1	1 : 1 randomized

Outline of a Simple Evolutionary Algorithm



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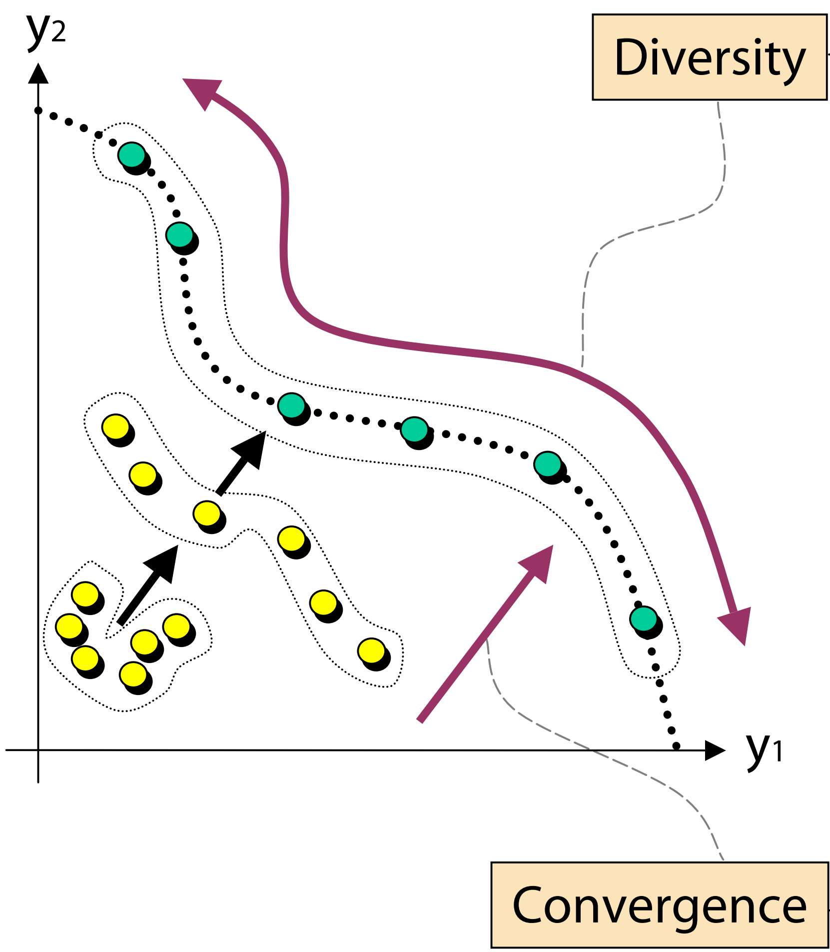
Issues in EMO



- How to maintain a diverse Pareto set approximation?

  ❷ **density estimation**

- How to prevent nondominated solutions from being lost?

  ❸ **environmental selection**

- How to guide the population towards the Pareto set?

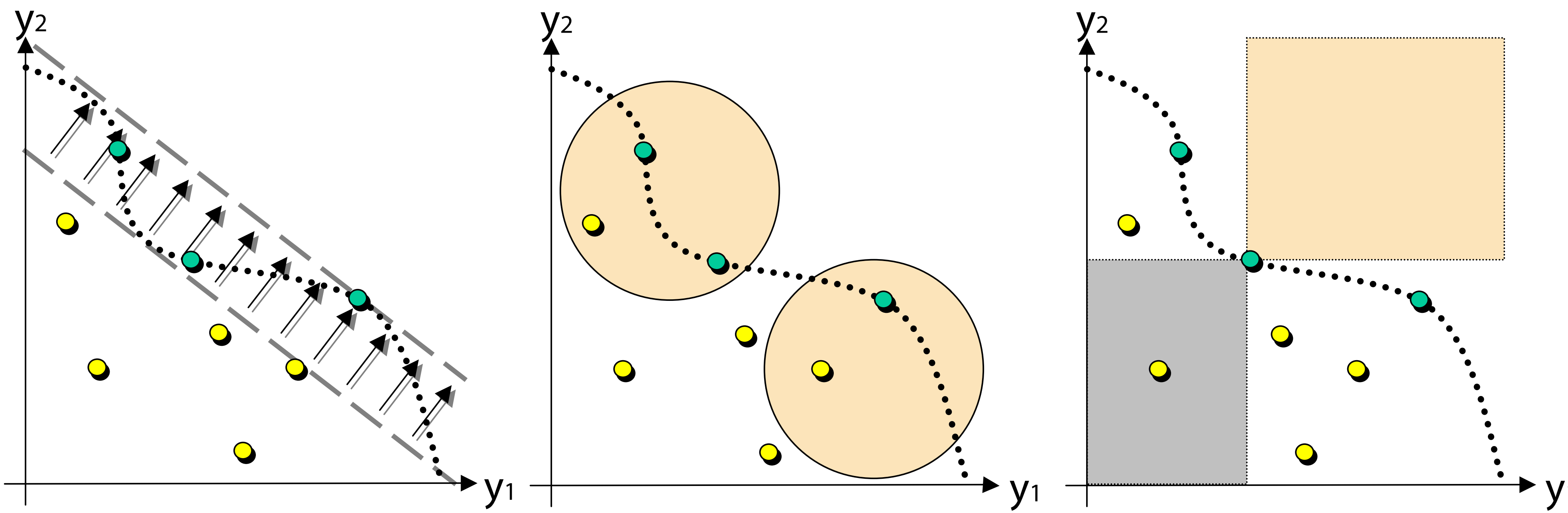
  ❶ **mating selection**

Fitness Assignment Strategies

aggregation-based
weighted sum

criterion-based
VEGA

dominance-based
SPEA2



parameter-oriented
scaling-dependent

set-oriented
scaling-independent

Dominance-Based Ranking

Types of information:

• **dominance rank**	by how many individuals is an individual dominated?
• **dominance count**	how many individuals does an individual dominate?
• **dominance depth**	at which front is an individual located?

Examples:

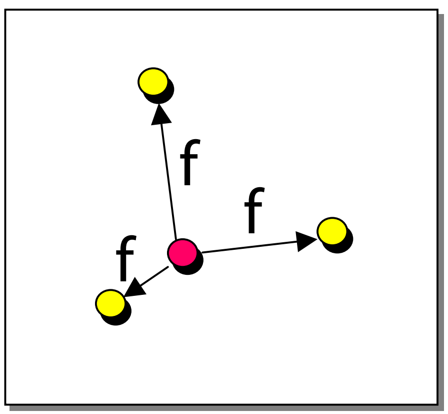
• *MOGA, NPGA*	dominance rank
• *NSGA/NSGA-II*	dominance depth
• *SPEA/SPEA2*	dominance count + rank

Density estimation techniques: *[Silverman: 1986]*

Kernel

MOGA

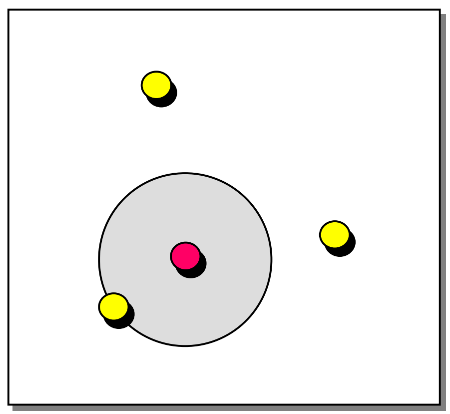
density estimate
=
sum of f values where f is a function of the distance



Nearest neighbor

SPEA2

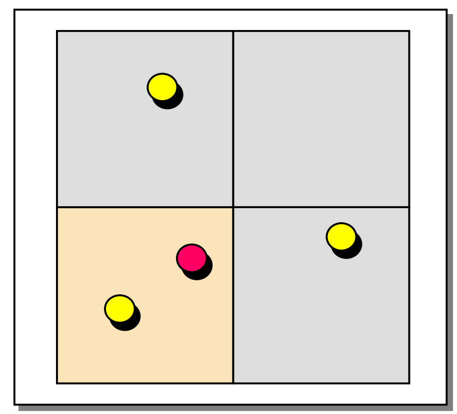
density estimate
=
volume of the sphere defined by the nearest neighbor



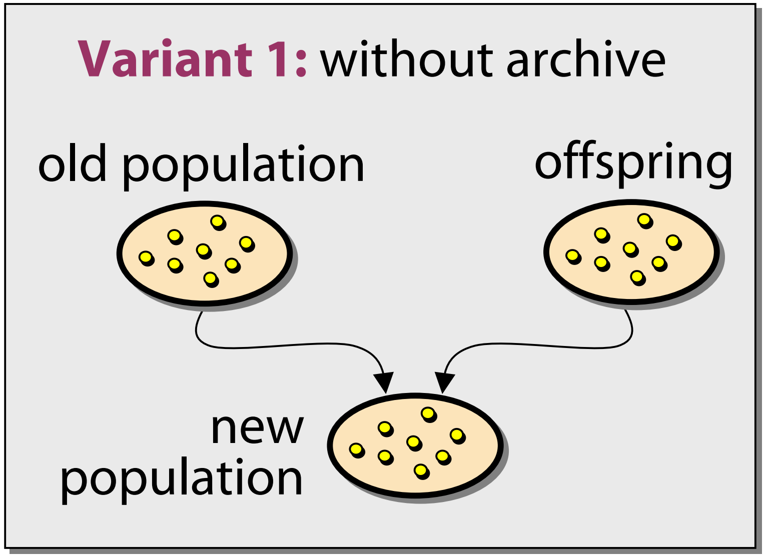
Histogram

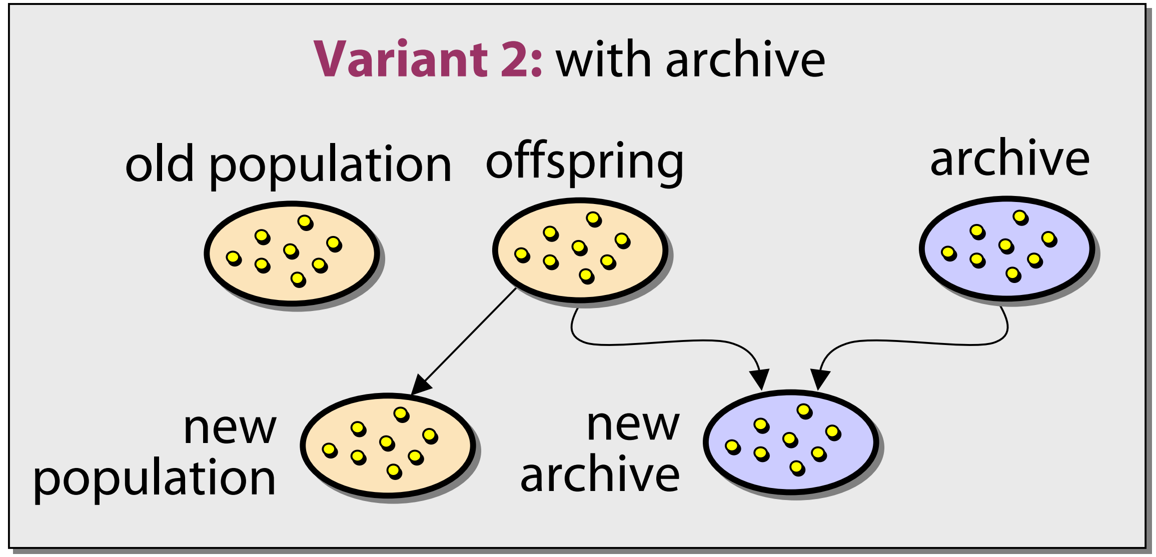
PAES

density estimate
=
number of solutions in the same box



Environmental Selection





Selection criteria:

- **Dominance:** only nondominated solutions are kept
- **Density**: less crowded regions are preferred to crowded regions
- **Time**: old archive members are preferred to new solutions
- **Chance:** each solution has the same probability to enter the archive

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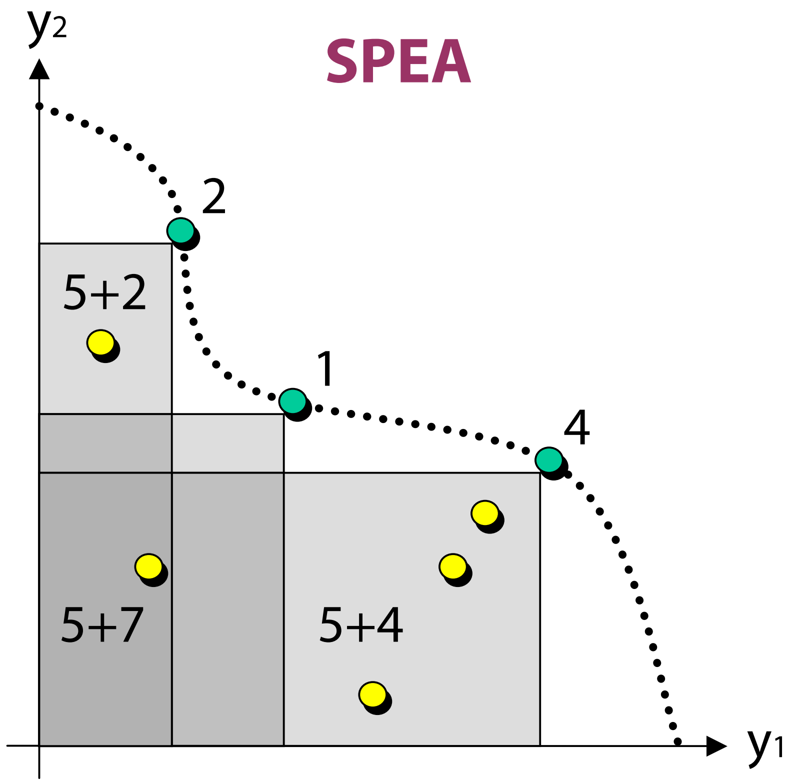
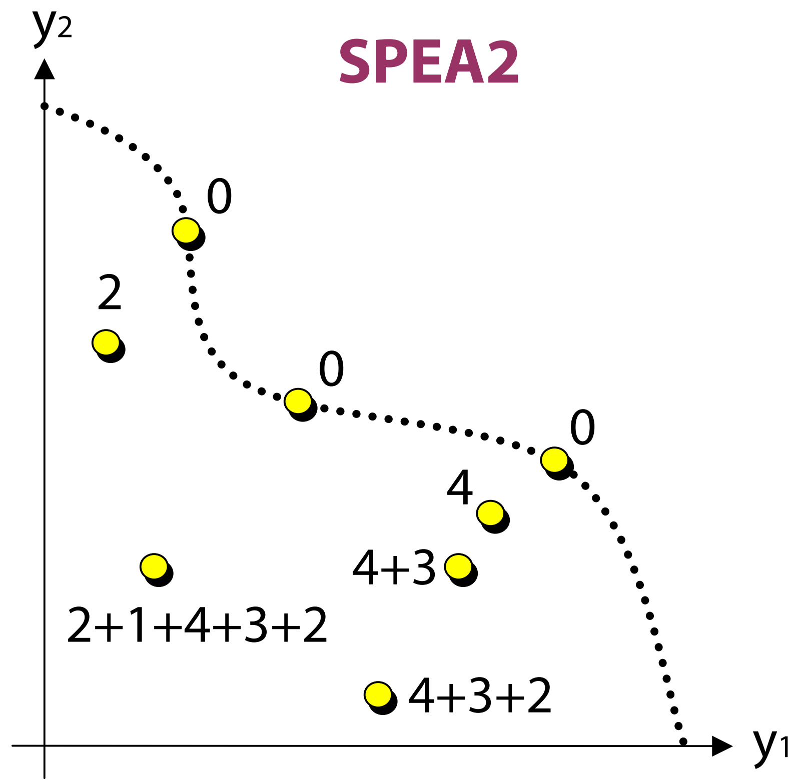
SPEA2 Algorithm

Step 1:	Generate initial population P_0 and empty archive (external set) A_0. Set $t = 0$.
Step 2:	Calculate fitness values of individuals in P_t and A_t.
Step 3:	A_{t+1} = nondominated individuals in P_t and A_t. If size of $A_{t+1} > N$ then reduce A_{t+1}, else if size of $A_{t+1} < N$ then fill A_{t+1} with dominated individuals in P_t and A_t.
Step 4:	If $t > T$ then output the nondominated set of A_{t+1}. Stop.
Step 5:	Fill mating pool by binary tournament selection with replacement on A_{t+1}.
Step 6:	Apply recombination and mutation operators to the mating pool and set P_{t+1} to the resulting population. Set $t = t + 1$ and go to Step 2.

[Zitzler, Laumanns, Thiele: 2001]

Pareto Fitness Assignment





SPEA

- S (strength) = #dominated solutions
- R (raw fitness) = N + $\sum$ strengths of dominators

SPEA2

- S (strength) = #dominated solutions
- R (raw fitness) = $\sum$ strengths of dominators

Diversity Preservation

Density Estimation

k-th nearest neighbor method:

- Fitness = R + $\underbrace{1 / (2 + D_k)}_{< 1}$
- D_k = distance to the k-th nearest individual
- $k = \sqrt{\text{popsize} + \text{archivesize}}$

Truncation

Incremental approach:

- Remove individual A for which A $<_d$ B for all individuals B
- B $<_d$ A iff:
  - D_k identical for A and B for all k
  - D_k of A greater than D_k of B for a particular k and identical for smaller k

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Performance Assessment: Approaches

❶ **Theoretically (by analysis):** difficult

- Limit behavior

  "Is the Pareto set found, if there are unlimited run-time resources?"

- Run-time analysis

  "How long does it take to generate the Pareto set with high probability?"

❷ **Empirically (by simulation):** standard

Limit Behavior

Basic assumptions:

- Every solution can be generated from every other solution by mutation
- The number of iterations t goes to infinity ($t \longrightarrow \infty$)

Studies:

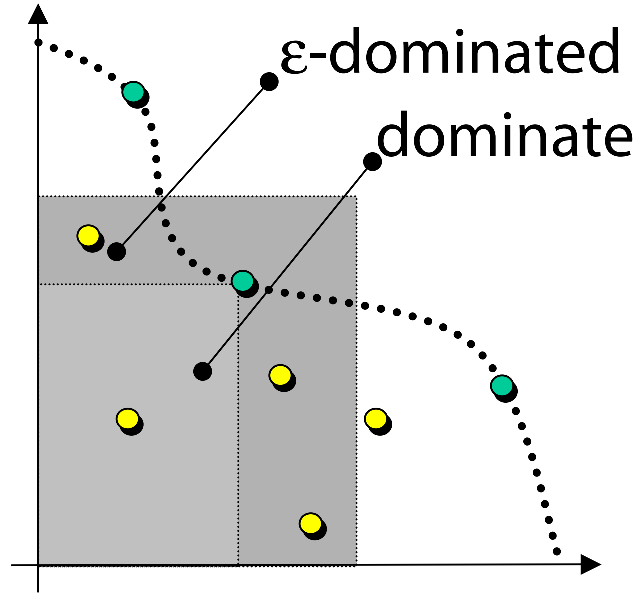
- Convergence: *[Hanne: 1999][Rudolph, Agapie: 2000]*
- Diversity: e.g., *[Knowles, Corne: 2000][Deb et al.: 2001]*
- Convergence + diversity:
  - Unlimited memory resources *[Rudolph and Agapie: 2000]*
  - Limited memory resources *[Laumanns et al.: 2002]*

Epsilon Dominance

Definition 1: ε-Dominance

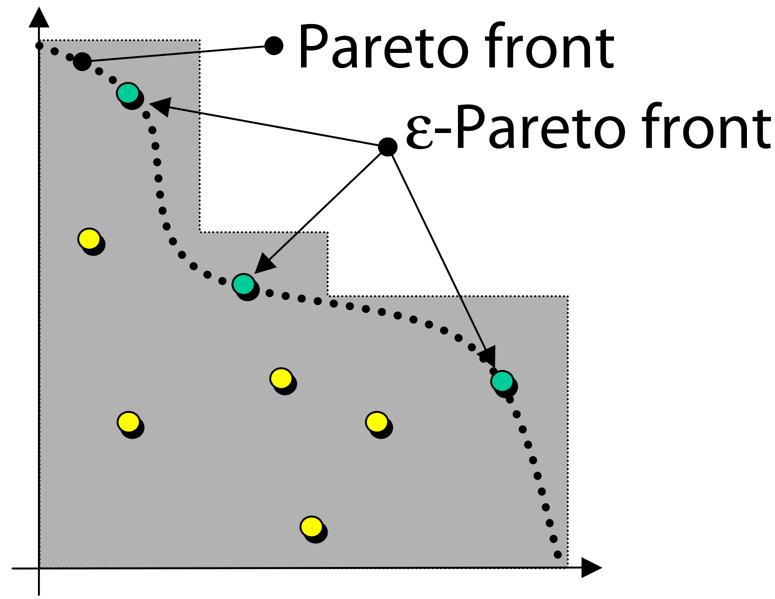
A ε-dominates B iff $\varepsilon \cdot f(A) \geq f(B)$

(known since 1987)



Definition 2: ε-Pareto set

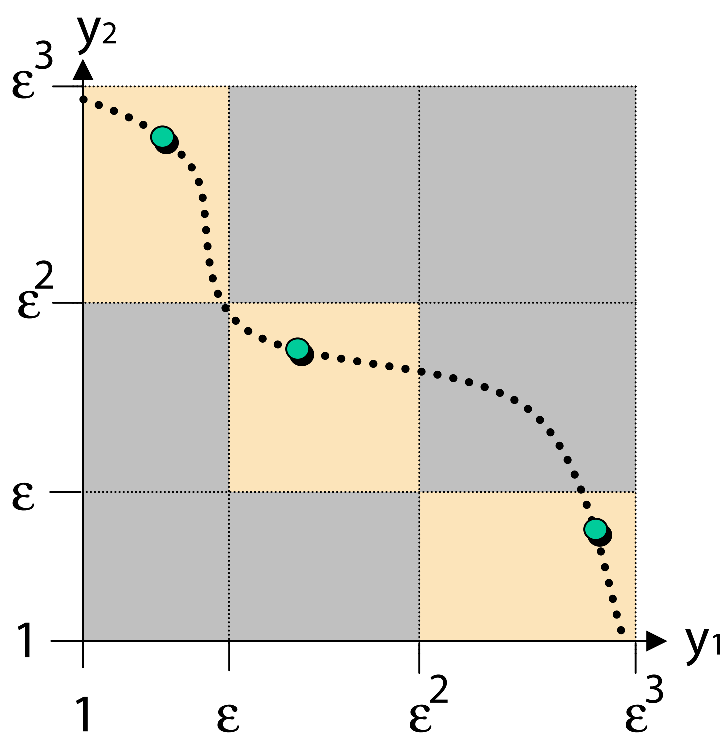
subset of the Pareto set which ε-dominates all Pareto-optimal solutions



Achieving Convergence and Diversity

Goal: Maintain ε-Pareto set

Idea: ε-grid, i.e. maintain a set of nondominated boxes (one solution per box)



Algorithm: (ε-update)

Accept a child if

❶ the corresponding box is not dominated by any box that contains an individual

AND

❷ any other individual in the same box is dominated by the new solution

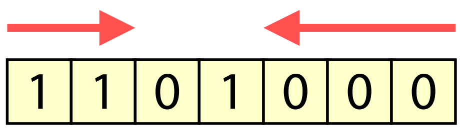
Run-Time Analysis of a Multiobjective EA

Basic question: *[Laumanns et al.: 2002]*

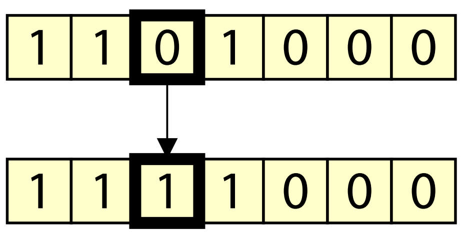
What is the worst case run-time of a multiobjective EA to find the Pareto set with high probability?

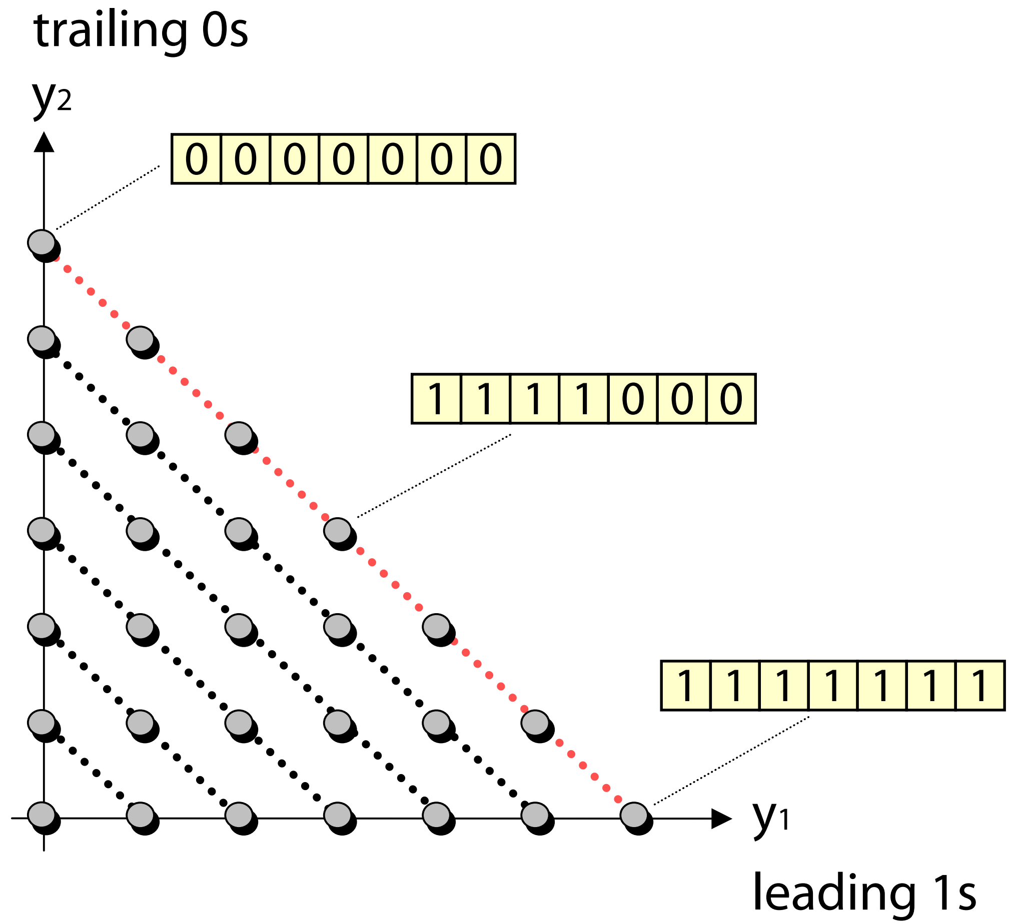
Scenario:

- **Problem:** leading ones, trailing zeros (LOTZ)

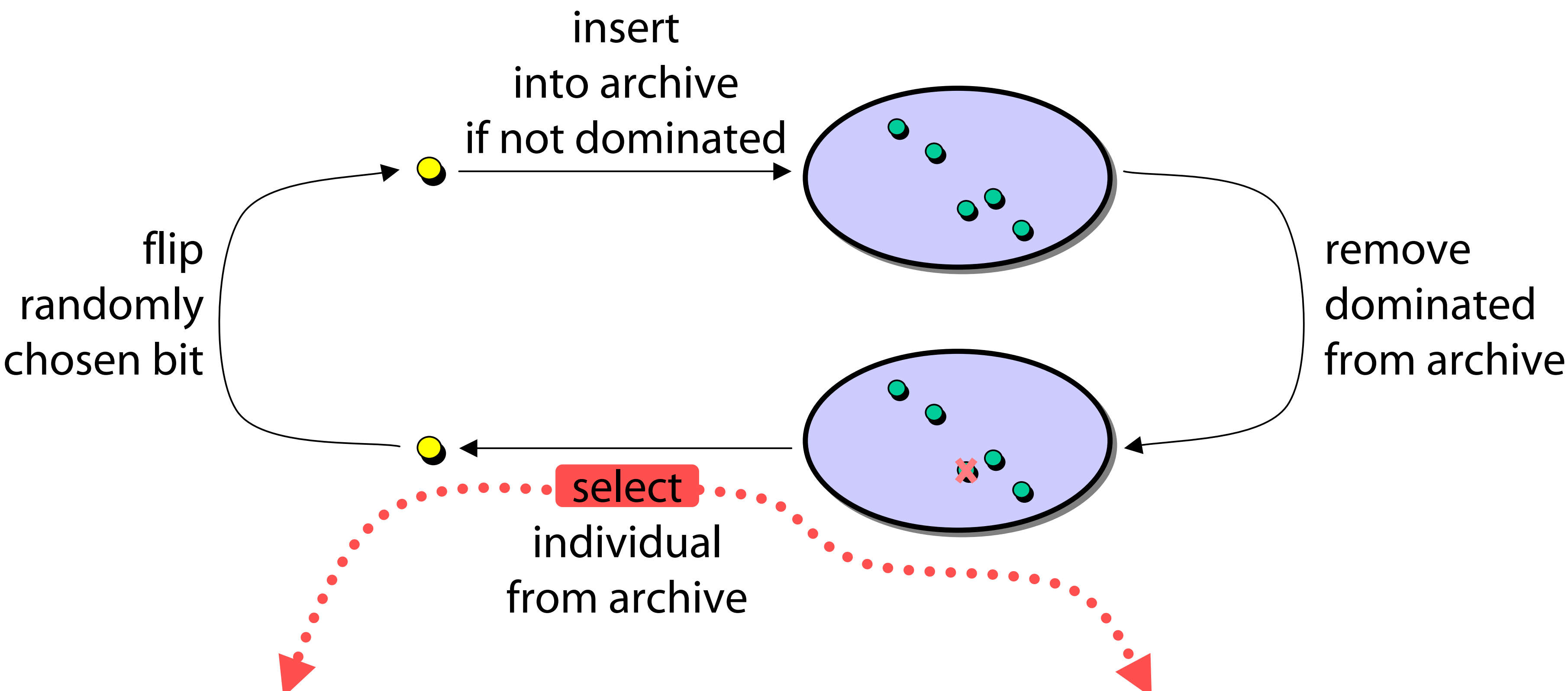


- **Variation:** single point mutation





Two Simple Multiobjective EAs



Variant 1: SEMO

Each individual in the archive is selected with the same probability (uniform selection)

Variant 2: FEMO

Select individual with minimum number of children (fair selection)

Results of Analysis

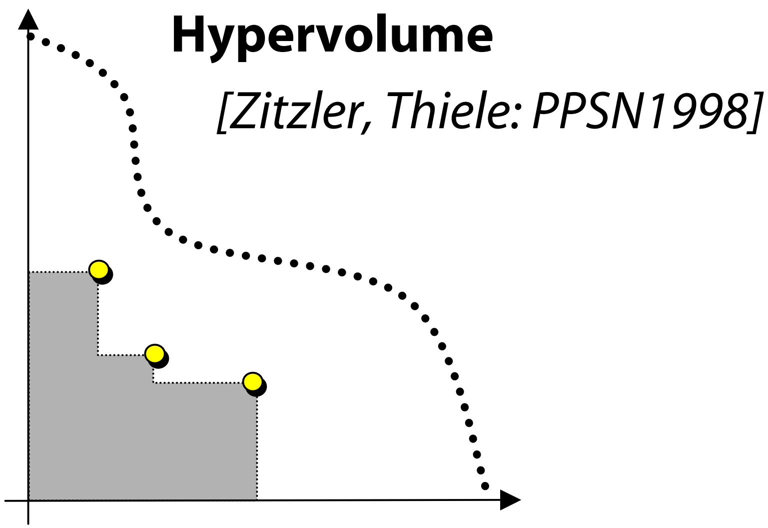
- Simple multiobjective EA with uniform selection (SEMO): $\Theta(n^3)$
  - To get to the Pareto front requires n^2 steps
  - To cover the entire front needs n^3 steps
- Simple multiobjective EA with fair selection (FEMO): $\Theta(n^2 \log n)$
  - Fair selection helps to spread over the Pareto front
- Multistart single-objective optimizer: $\Omega(n^3)$
  - In average, one out of *n* mutations successful
  - To get to the Pareto front, *n* successful mutations needed
  - Overall *n* Pareto-optimal solutions have to be found

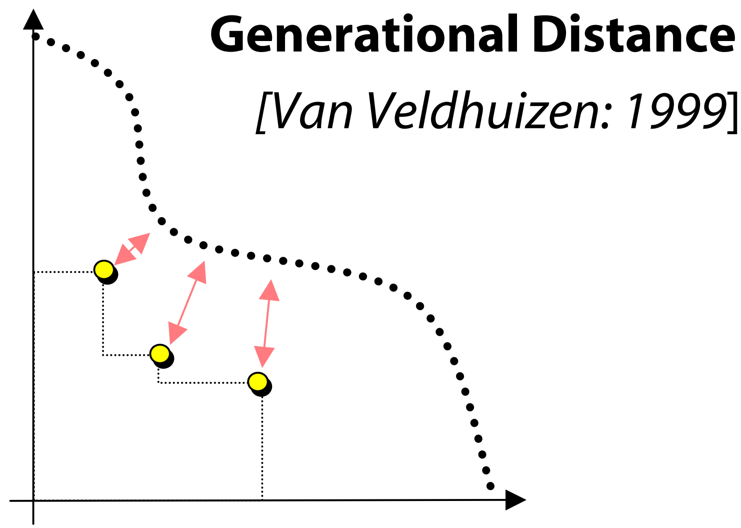
multiobjective EA faster than multistart strategy

Empirical Performance Assessment

Issues: quality measures, statistical testing, benchmark problems, visualization, …

Popular approach: unary quality measures

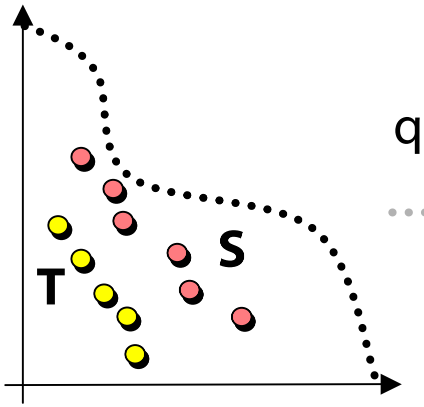




- Assign each outcome a *real number*
- Outcomes are compared by comparing the corresponding *values*

Quality Measures: Theoretical Results I

Basic question: Can we say on the basis of the quality measures *whether* or *that* an algorithm outperforms another?



application of quality measures

	S	T
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

There is no combination of unary quality measures such that **S** is better than **T** in all criteria is equivalent to **S** dominates **T**

Unary quality measures usually do not tell that **S** dominates **T**; at maximum that **S** does not dominate **T**

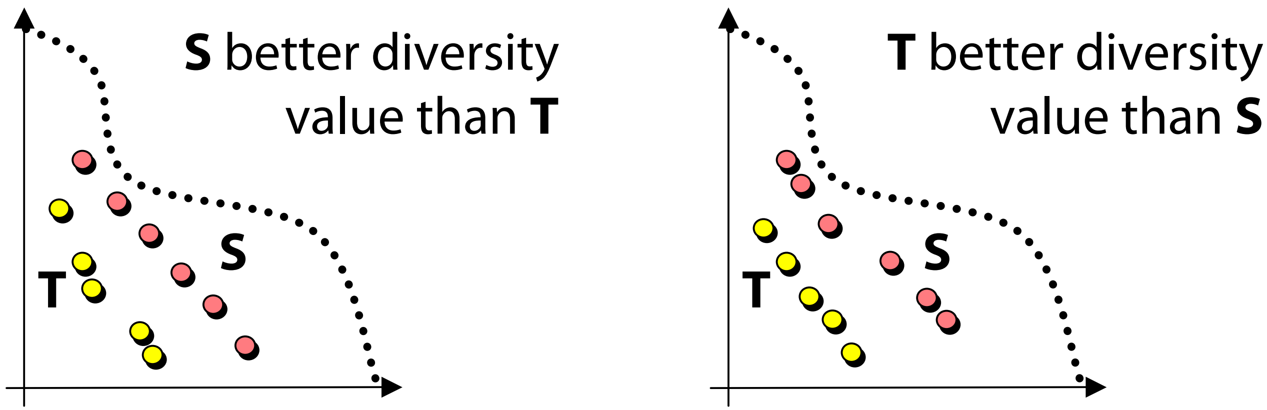
[Zitzler et al.: 2002]

Quality Measures: Theoretical Results II

Many popular quality measures are not compliant with the dominance relation

[Hansen, Jaszkiewicz: 1998][Knowles, Corne: 2002][Zitzler et al.: 2002]

Example: diversity measures

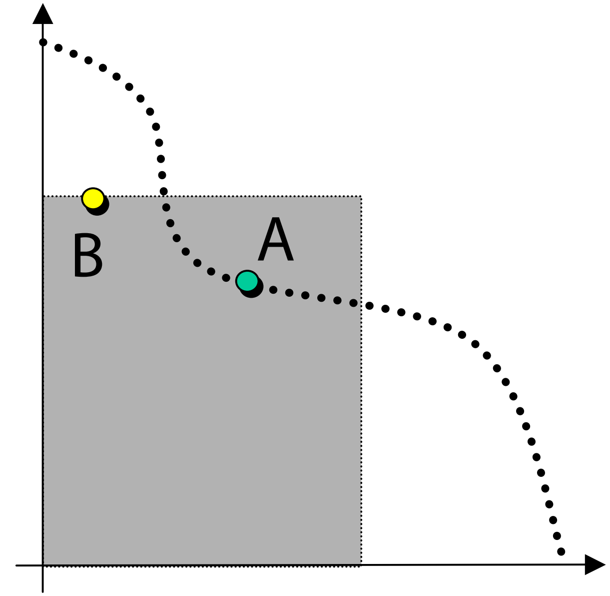


Needed: appropriate binary quality measures that indicate *whether* an outcome dominates another, e.g., ε-measure

Epsilon Quality Measure

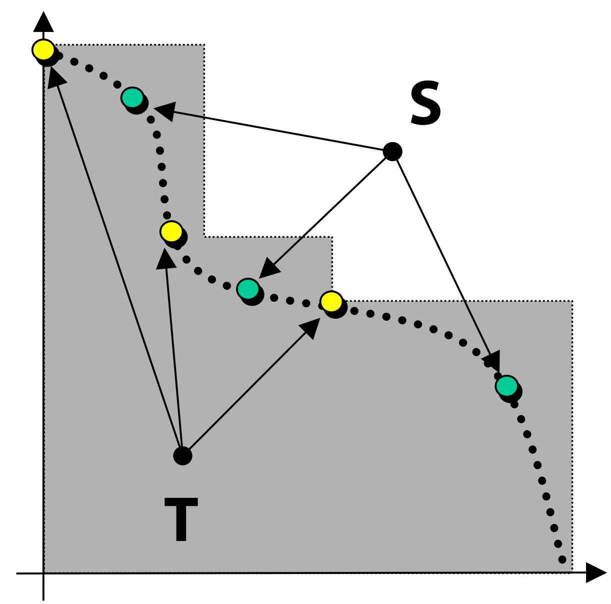
Definition 3: single solutions

$I_\varepsilon(A,B)$ = minimum ε such that A ε-dominates B



Definition 4: sets of solutions

$I_\varepsilon(\mathbf{S},\mathbf{T})$ = minimum ε such that each solution in **T** is ε-dominated by at least one solution in **S**



[Zitzler et al.: 2002]

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Implementation: Current Situation

Application engineer

- knowledge in the algorithm domain necessary
- state-of-the-art algorithms get more and more complex
- many algorithms

Algorithm designer

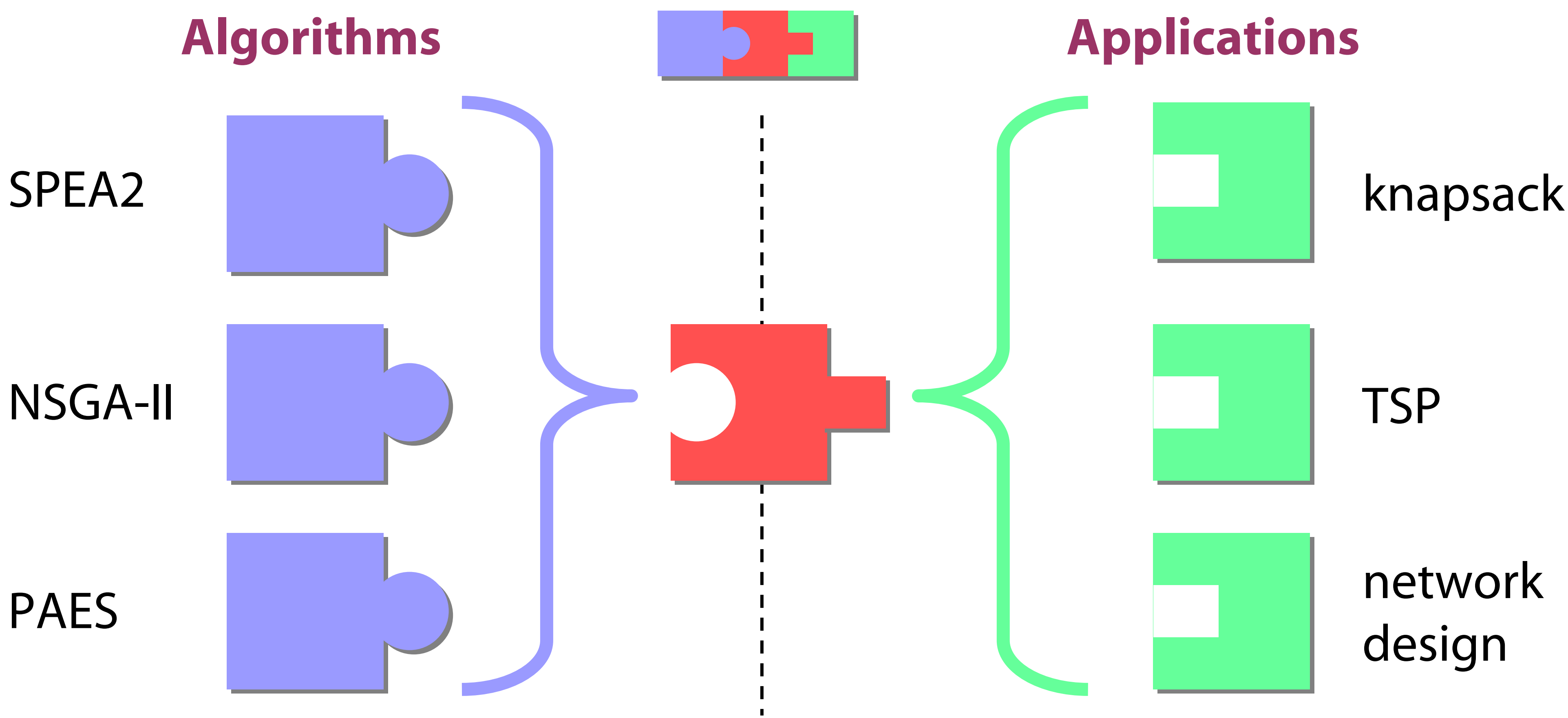
- comparison to competing algorithms mandatory
- tests on various benchmark problems necessary
- algorithms and applications become increasingly complex

high implementation effort / risk of implementation errors

Programming libraries:

+ valuable tools to tailor a particular technique to a specific application
− exchange of optimization algorithm or application still difficult

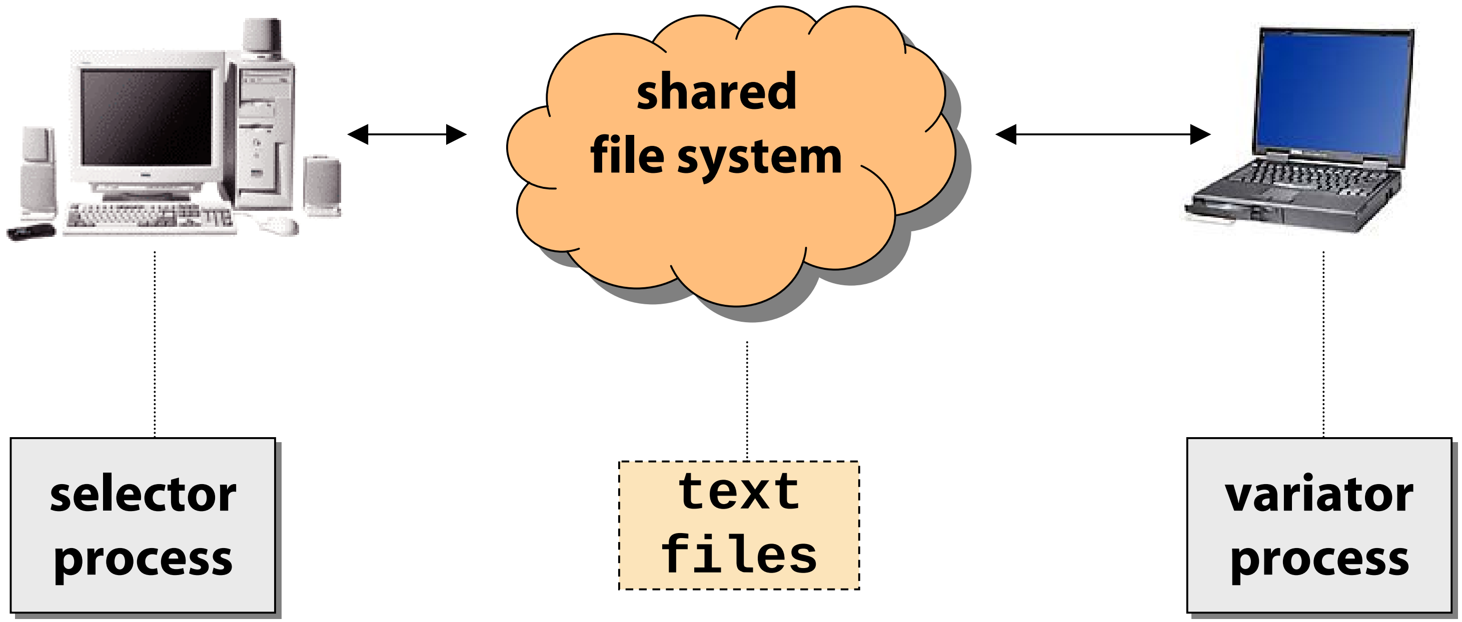
The Concept of PISA



text-based

Platform and programming language independent **I**nterface for **S**earch **A**lgorithms *[Bleuler et al.: 2002]*

PISA: Implementation



application independent:

- mating / environmental selection
- individuals are described by IDs and objective vectors

handshake protocol:

- state / action
- individual IDs
- objective vectors
- parameters

application dependent:

- variation operators
- stores and manages individuals

Conclusions

Why using an evolutionary algorithm?

- **Flexibility:** problem formulation can be easily modified / extended (minimum requirements)
- **Multiple objectives:** the solution space can be explored in a single optimization run
- **Feasibility:** EAs are applicable to complex and huge search spaces

Why multiobjective optimization?

- **Robustness:** aggregation of several objectives into a single one requires setting of parameters
- **Confidence:** it is easier to select a solution if alternatives are known

Main application of EMO: design space exploration

More on EMO

Links:

- EMO mailing list:
  http://w3.ualg.pt/lists/emo-list/
- EMO bibliography:
  http://www.lania.mx/~ccoello/EMOO/
- PISA website:
  http://www.tik.ee.ethz.ch/pisa/

Events:

- Conference on Evolutionary Multi-Criterion Optimization (EMO 2003), April 8-11, 2003, Algarve, Portugal:
  http://conferences.ptrede.com/emo03/

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