



# Quantum Neural Network model for Token allocation for Course Bidding

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## ABSTRACT

Quantum computer has shown the advantage over the classical computer to solve some problems using the laws of quantum mechanics. With a combination of knowledge of machine learning and quantum computing, Quantum neural networks adapted the concept from classical neural networks and apply parameterized quantum gates as neural network weights. In this paper, we present an application of quantum neural networks with real-world data to predict token price used in a course bidding system. The experiments were carried out on the Qiskit quantum simulator. The result shows that quantum neural networks can achieve a good prediction result compared to the classical neural network. The best model configuration has the lowest RMSE 6.38%. This approach opens an opportunity to explore the benefit of quantum machine learning in many research fields in the future.

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## 1. INTRODUCTION

Machine learning and Quantum computing are two research areas that have attracted considerable attention in recent years and have evolved into a new field known as Quantum machine learning [1]. Many research papers include [2, 3, 4, 5, 6] show the potential advantages such as speed up in training a model and in [7] has shown the power of using quantum neural networks to train machine learning model. One promising way to implement Quantum algorithms in the Noisy Intermediate Scale Quantum (NISQ) [8, 9] is using a technique call variational quantum circuits or trainable quantum circuits as a machine learning model [10, 11, 12, 13, 14].

Quantum Neural Networks take advantage of a quantum computer using quantum mechanics such as superposition, entanglement, on quantum bits to perform the calculation [15]. The motivation behind this research is to present the application of Quantum Neural networks with real-world data and practical challenges that are yet to be solved by using the new method on near-term quantum devices.

In this paper, we propose a quantum computing method to predict the token price to suggest and provide information to users in a course bidding system.

## 2. QUANTUM COMPUTING

In the classical computer, the basic unit of information is known as Bit but in the quantum computer, the smallest unit is the qubit or quantum bit. In a classical system, a bit would have to be in one state (0 or 1). However, a quantum bit may be in a superposition of both states simultaneously. This means that the probabilities of measuring 0 or 1 for a qubit are in general neither 0.0 nor 1.0.

An important distinguishing feature between qubits and classical bits is that multiple qubits can reveal quantum entanglement. Quantum entanglement is a quantum mechanical phenomenon when two or more qubits can interact, communicate, or correlate with each other no matter how far apart they are in space, their states remain linked and share a common quantum state. Measurements of one of the qubit automatically influence the other and once we have knowledge of one quantum state, we automatically know the quantum state of any entangled qubits

### 2.1 Quantum circuit model of computation

A quantum circuit is a prescription for quantum operations we will perform on some set of the quan-

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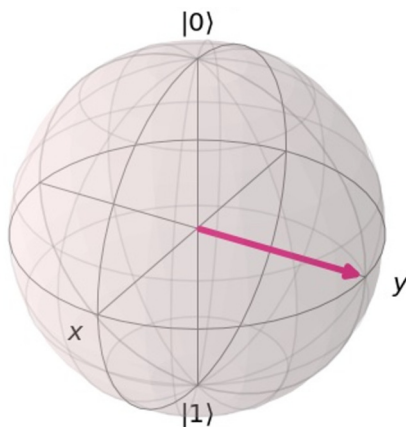
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tum state similar to classical circuits in which a computation is a sequence of quantum gates to represent the quantum program or algorithm.

As we will see the quantum circuits are built out of a small set of gates. We will list a few of the most important gates that we will encounter in this paper in Table 1. Quantum gates are represented by unitary matrices and their circuit form.

### 2.1.1 Single Qubit gate

A single qubit gate is a gate that acts on a single qubit. There are some common quantum gates such as Hadamard Gate and Pauli gates (X, Y, Z) in most quantum circuit. Hadamard Gate is a basic quantum gate that maps the basis state  $|0\rangle$  or  $|1\rangle$  to superposition state. Pauli gates are a single qubit gate rotation through  $\pi$  radians around the X, Y, and Z axes of the Bloch sphere showing in Fig. 1. The Pauli-X gate is the quantum equivalent of the NOT gate for classical computers.



**Fig. 1:** Bloch sphere.

### 2.1.2 Multi Qubit gate

A Multi-qubit gate is a quantum gate that acts on multiple qubits and utilizes the true power of quantum computing through the interactions between qubits or entanglement between qubits.

CNOT or Controlled NOT gate operates on 2 qubits by using the first qubit as a control bit and will flip the second qubit if and only if the first qubit is  $|1\rangle$ . The quantum truth table of the CNOT gate is shown in Table 2.

### 2.1.3 Parameterized gate

The parameterized gate or rotation operator gate is a special quantum gate that can specify the rotation angle  $\theta$ . The Rx, Ry, and Rz are similar to Pauli-X, Y, Z gates but we can specify the rotation angle instead of  $\pi$  radians in the Pauli gate.

## 3. QUANTUM NEURAL NETWORK

A Quantum neural network is an algorithm designed for execution on a NISQ device by combining quantum computers with classical computers. It is a subclass of variational quantum algorithms using trainable quantum circuits as a machine learning model. Quantum computers will be used as hardware accelerators co-working with a classical computer.

### 3.1 Variational quantum algorithms (VQAs)

A quantum algorithm is an algorithm or step-by-step procedure for solving a certain problem by using quantum superposition and quantum entanglement to perform a calculation on a quantum computer.

Mostly we use a quantum circuit model of computation to perform a quantum algorithm. The well-known quantum algorithm is Shor's algorithm for solving Integer factorization or Prime decomposition and Grover's algorithm for Unstructured search algorithm or Brute-force searching based on amplitude amplification.









In the NISQ era, VQAs are algorithms that allow near-term quantum advantage, comprised of an iterative quantum-classical optimization loop between a classical computer and a quantum computer. In each iteration the classical computer sends the set of quantum logic gate parameters  $\theta$  to the quantum circuit then the circuit was executed on the quantum device. The estimated expectation value is sent back to the classical computer where the classical optimizer is running and suggests a new set of parameters for the subsequent iteration to minimized or maximized the cost function. The well-known algorithm using the concept of variational quantum algorithms is Quantum Approximate Optimization Algorithm (QAOA) [16, 17].

### 3.2 Parameterized quantum circuits as machine learning

In [10] Marcello Benedetti *et al.*, and [13] Yuxuan Du *et al.*, propose parameterized quantum circuit as a machine learning model. Maria Schuld [5][14] propose Circuit-centric quantum classifiers for use in supervised learning by using a technic called Low-depth variational quantum algorithm to train a machine learning model. It is a hybrid loop of calculation between the quantum computer and classical computer (Hybrid quantum-classical gradient descent).

The greatest strength of a Low-depth variational quantum algorithm is using fewer learnable parameters or parametrized gates compared to other algorithms. The number of parameters grows in Polylogarithmic by the input dimension so that the quantum circuit is smaller and can work on the NISQ device.

**Table 1:** Example of Quantum gates.

Name	Circuit form	Matrices representation
Hadamard Gate	q —  —	$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Pauli-X gate	q —  —	$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y gate	q —  —	$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z gate	q —  —	$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
CNOT gate		$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Rx gate	q1 —  —	$R_x(\theta) = \begin{bmatrix} \cos(\theta/2) & -i \sin(\theta/2) \\ -i \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$
Ry gate	q0 —  —	$R_y(\theta) = \begin{bmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$
Rz gate	q2 —  —	$R_z(\theta) = \begin{bmatrix} \exp(-i\theta/2) & 0 \\ 0 & \exp(i\theta/2) \end{bmatrix}$

**Table 2:** The quantum truth table of the CNOT gate.

Before		After	
Control	Target	Control	Target
0⟩	0⟩	0⟩	0⟩
0⟩	1⟩	0⟩	1⟩
1⟩	0⟩	1⟩	1⟩
1⟩	1⟩	1⟩	0⟩

Student_Id	Course_Id	Course_name_x	Year	Limit	Token	Enrolled	Regstatus	Refund	Forcerefund
0	55 xxxxxxxx	444	COMP SECURITY	2015	40	260001	1	11	0
1	53 xxxxxxxx	444	COMP SECURITY	2015	40	50000	0	20	1
2	55 xxxxxxxx	444	COMP SECURITY	2015	40	260000	1	11	0
3	56 xxxxxxxx	444	COMP SECURITY	2015	40	270000	1	11	0
4	55 xxxxxxxx	444	COMP SECURITY	2015	40	302991	1	11	0

**Fig.2:** The dataset from a course bidding system.

	all_max	all_min	all_mean	enrolled_max	enrolled_min	enrolled_mean	unenrolled_max	unenrolled_min	unenrolled_mean	Token
all_max	1.00	0.14	0.78	0.97	0.57	0.76	0.68	0.24	0.51	0.65
all_min	0.14	1.00	0.15	0.14	0.28	0.14	0.12	-0.01	0.06	0.12
all_mean	0.78	0.15	1.00	0.80	0.80	0.97	0.70	0.01	0.43	0.84
enrolled_max	0.97	0.14	0.80	1.00	0.60	0.78	0.59	-0.00	0.31	0.68
enrolled_min	0.57	0.28	0.80	0.60	1.00	0.88	0.63	-0.08	0.37	0.76
enrolled_mean	0.76	0.14	0.97	0.78	0.88	1.00	0.70	-0.05	0.39	0.86
unenrolled_max	0.68	0.12	0.70	0.59	0.63	0.70	1.00	0.39	0.72	0.60
unenrolled_min	0.24	-0.01	0.01	-0.00	-0.08	-0.05	0.39	1.00	0.87	-0.04
unenrolled_mean	0.51	0.06	0.43	0.31	0.37	0.39	0.72	0.87	1.00	0.34
Token	0.65	0.12	0.84	0.68	0.76	0.86	0.60	-0.04	0.34	1.00

**Fig.3:** Correlation score.

### 3.3 Second-order Pauli-Z evolution circuit

Second-order Pauli-Z evolution circuit (ZZFeaturemap) is a quantum feature map circuit that transforms classical input data to a quantum state. It takes a classical data point and translates it into a set of parameterized quantum gates in a quantum circuit. ZZFeaturemap is based on the Pauli Expansion circuit which uses the RZ-gate as a parameterized gate and CNOT gate to control entanglement and interference between qubits.

Entanglement in the system is achieved via CNOT gates. The entanglement strategies are full and linear, corresponding to the full (or all-to-all) and linear (or next-neighbor coupling) entangler maps.

### 3.4 Quantum neural network model

QNN is inspired by a classical neural network that tries to mimic the structure of a classical neural network and use the parameterized quantum gates as the weights within a neural network. The training data are encoded into a quantum state via the feature map circuit. The number of qubits used depends on the training data attributes, one attribute per qubit. The feedforward and hidden layer are in the form of the variational quantum circuit. The backpropagation part measures all qubit output and calculates through loss function minimization. The goal is to optimize over a parameterized circuit, then set optimized parameters back to the variational quantum circuit.

## 4. TOKEN ALLOCATION FOR COURSE BIDDING

In paper [18] presents previous work about the Machine learning model used to predict the token price for allocation to course through a course bidding system. Three machine learning models are compared: Decision Tree, Random Forest, and Artificial Neuron Network (ANN). The dataset is from a course bidding system showing in Fig. 2 and was pre-processed into eight input variables with the highest correlation score and one output variable. ANN is the best performing method to predict token price, with two hidden layers and one output layer, in each hidden layer has eight neurons fully connected.

The result of the experiment shows that ANN is the best method with the lowest RMSE 3.98% over Decision Tree with RMSE 4.18% and Random Forest with RMSE 4.13%. This result inspired us to implement ANN in a new proposed quantum machine learning method to demonstrate the ability to use QNN with the real-world data set.

## 5. METHOD

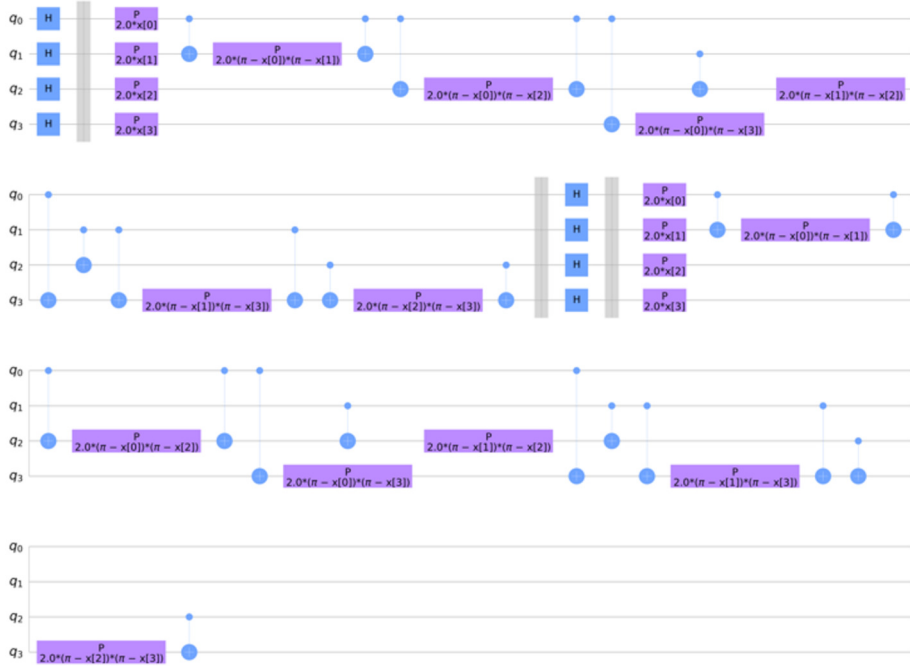
In this paper, we focus on implementing the QNN model by using a quantum simulator from Qiskit [19]. The quantum computer is very difficult to simulate classically and the resource required to grow exponentially with the number of qubits or the depth of the circuit. From this limitation, we limit the number of the qubit to only four qubits. This means the input for this model needs to be select from the most important by highest correlation score in Fig. 3, four attributes shown in Table 3 were selected and data point was filtered by course interesting in which values more than two are used.

**Table 3:** Input and output variable.

Input	Output
course interesting	Token price
all_mean	
enrolled_min	
enrolled_mean	

### 5.1 Data Encoding

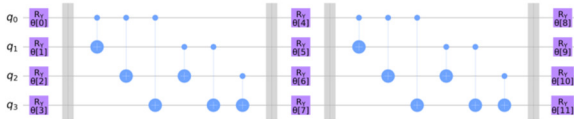
The first step is to translate classical data into the quantum state. We use a Second-order Pauli-Z evolution circuit (ZZFeatureMap) developed in [20] with four qubits and two repeated circuits. Hadamard gate applies on each qubit, followed by a layer of RZ-gates used to encode data and CNOT-gates on every pair of a qubit. With full entanglement, each qubit is entangled with all the others. The output of the feature map circuit is quantum state and will be used as input of the quantum neural network. ZZFeatureMap circuit is shown in Fig. 4.



**Fig.4:** Second-order Pauli-Z evolution circuit (ZZFeatureMap) with two repeated circuits, Hadamard gate applies on each qubit, followed by a layer of RZ-gates and CNOT-gates on every pair of a qubit.

**5.2 QNN Model**

For QNN, we used RealAmplitudes variational circuit shown in Fig. 5, The circuit consists of 4 qubits with Full entanglement. The layer of parameterized RY-gates is applied to each qubit and used as neural network weights. Increasing the depth of the variational circuit means we have more trainable parameters in the model. The number of weight or trainable parameters can be calculated as  $d = (D+1)S$ , where S is an input size or the number of qubits and D is the depth of the circuit (the number of the repeated circuit).

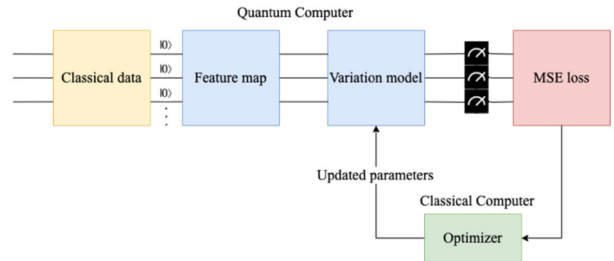


**Fig.5:** RealAmplitudes circuit with two repeated circuits, Layer of RY-gates followed by CNOT-gates is applied on every pair of a qubit. The circuit has a total of 12 trainable parameters.

**5.3 Model Training**

In this paper, we experiment on the number of repeated circuits to find the best model structure. We trained the model for 500 iterations on circuit depth range from 2-5, ADAM [21] optimizer with learning rate 0.001 and 100 iterations on circuit depth range from 4-7. ADAM optimizer with learning rate 0.1 is used to shorten the training time. Both use the

same MSE loss function. The overview of the training process is shown in Fig. 6-7 and the training loss values are plot in Fig.8-9



**Fig.6:** Overview the quantum neural network training process. Feature map and Variation model are executed on quantum computer and optimization is on a classical computer.

**6. RESULT**

The result of the QNN model prediction is shown in Table 4. We measure model performance by using RMSE on the testing data. Testing data was selected randomly for 30% of the samples.

Increasing the number of repeated circuits from 2-5, the model can perform better and RMSE values are decreasing dramatically. Increasing the number of repeated circuits to more than five, the model seems to be overfitting. RMSE values from testing data of repeated circuits 6-7 are very close to 5.

The best model configuration is five repeated circuits with 24 trainable gates, with a learning rate of

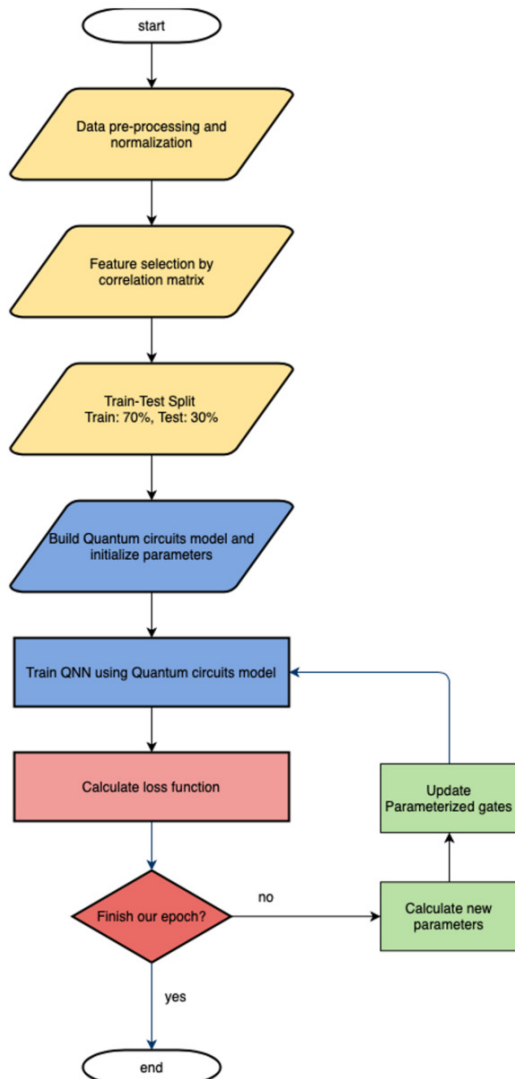


Fig. 7: Flow chart of the training process.

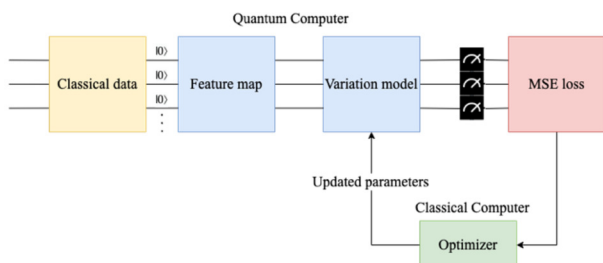


Fig. 8: ADAM optimizer with learning rate 0.001, Mean squared error loss as a function of training. The number of repeated circuit range from 2-5. We find that five repeated circuits have the lowest loss value.

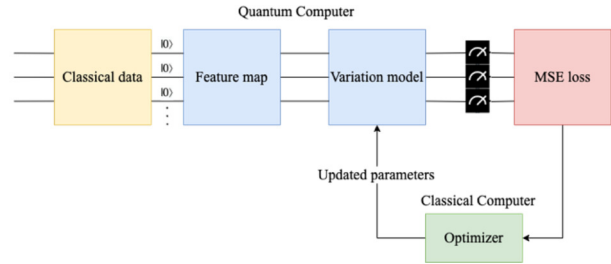


Fig. 9: ADAM optimizer with learning rate 0.1, Mean squared error loss as a function of training. The number of repeated circuit range from 4-7. We find that five repeated circuits still have the lowest loss value.

0.001 has the lowest RMSE at 7.8%, learning rate of 0.1 has RMSE at 6.38%. RMSE values from 500 iterations are plotted in Fig. 10.

Table 4: Experiment result.

Number of repeated circuits	Number of trainable gates	RMSE	
		$lr = 0.001$	$lr = 0.1$
2	12	0.0952	-
3	16	0.1198	-
4	20	0.1097	0.0691
5	24	0.0780	0.0638
6	28	-	0.0632
7	32	-	0.0633

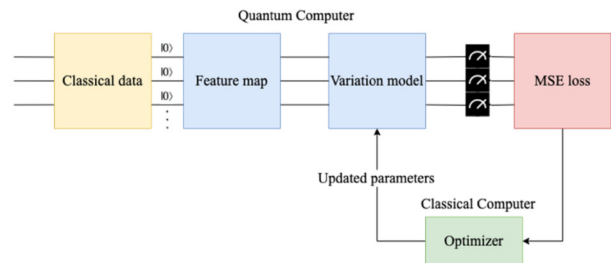


Fig. 10: ADAM optimizer with learning rate 0.001, RMSE on four different model. QNN Model with five repeated circuit has shown the lowest error.

## 7. CONCLUSION

In this paper, The result has shown that the Quantum Neural Network model can be trained and can perform regression tasks on real-world data set with a good result compared to the classical neural network. Our Quantum Neural network model with a limited input size of 4 and 24 weight parameters has the lowest RMSE 6.38% compared to the ANN model in paper [18] with 8 inputs and 153 weight parameters has the lowest RMSE 3.98%. The variational algorithms only employ shallow depth quantum circuits and can be implemented on noisy intermediate-scale quantum (NISQ) devices. It has shown the potential of using Quantum computers in Machine learning.

In addition, increasing repeated circuit or RY-gate is similar to increasing the number of nodes in a classical neural not only increases the capacity of a model network but also helps reduce training loss value and reduce the error of the predicted result.

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