



AUTOMATIC SPEECH RECOGNITION

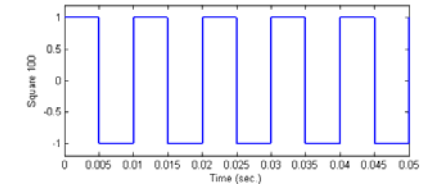
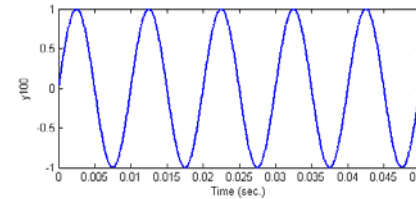
Lecture 2 & 3

Speech Signal Fundamentals



Signal

- Periodic Signal



- Non-periodic Signal
- Random Signal

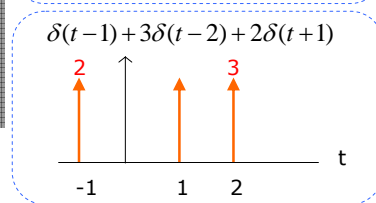
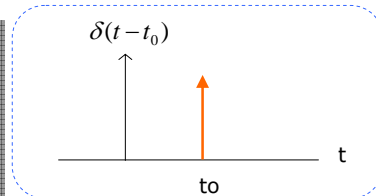


Impulse

$$\delta(t-t_0) = \begin{cases} \infty & ; t = t_0 \\ 0 & ; t \neq t_0 \end{cases}$$

and

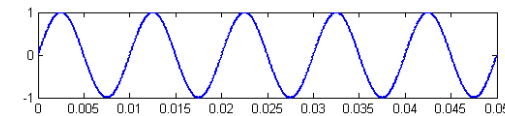
$$\int_{-\infty}^{\infty} \delta(t-t_0) dt = 1$$



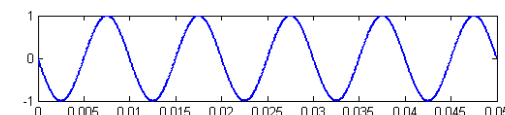
Sinusoidal Signal

$$y = A \sin(2\pi ft + \phi)$$

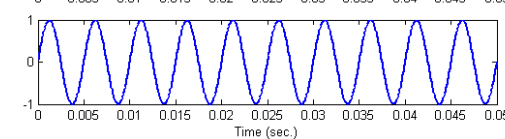
$f = 100\text{Hz.}$
 $\phi = 0$



$f = 100\text{Hz.}$
 $\phi = \pi$



$f = 200\text{Hz.}$
 $\phi = 0$



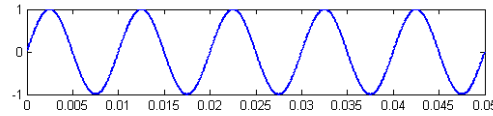
$A = 1$



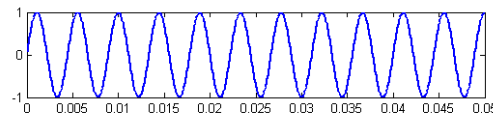
Linear Combination of Signals

$$s = a_1y_1 + a_2y_2 + \dots + a_Ny_N$$

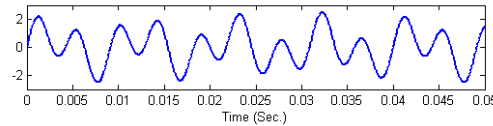
$$y_1 = \sin(2\pi(100)t)$$



$$y_2 = \sin(2\pi(225)t)$$

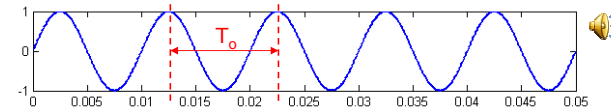


$$s = y_1 + 1.5y_2$$

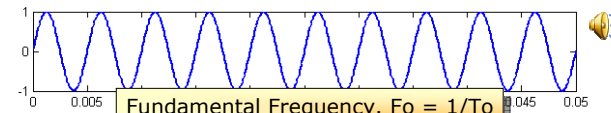


Fundamental Frequency and Harmonics

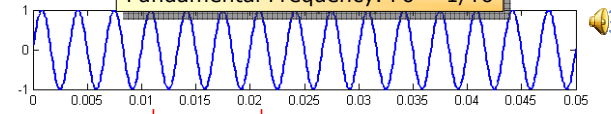
$$y_1 = \sin(2\pi(100)t)$$



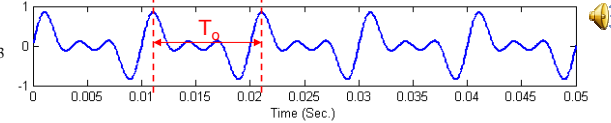
$$y_2 = \sin(2\pi(200)t)$$



$$y_3 = \sin(2\pi(300)t)$$

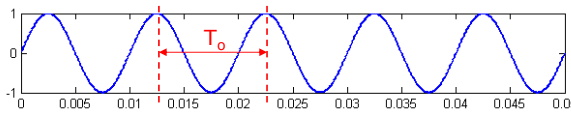


$$s = 0.3y_1 + 0.4y_2 + 0.3y_3$$

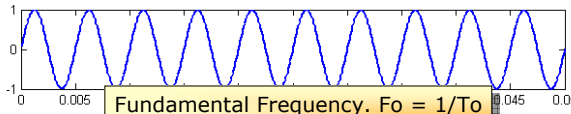


Fundamental Frequency and Harmonics

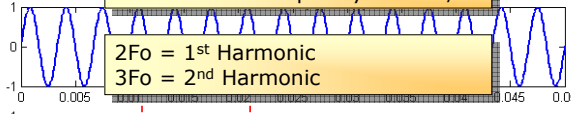
$$y_1 = \sin(2\pi(100)t)$$



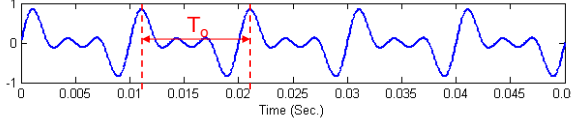
$$y_2 = \sin(2\pi(200)t)$$



$$y_3 = \sin(2\pi(300)t)$$

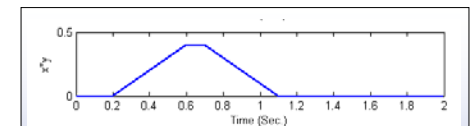
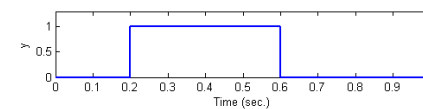
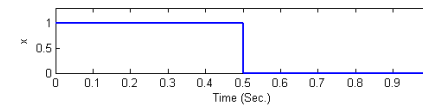


$$s = 0.3y_1 + 0.4y_2 + 0.3y_3$$



Convolution

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$$



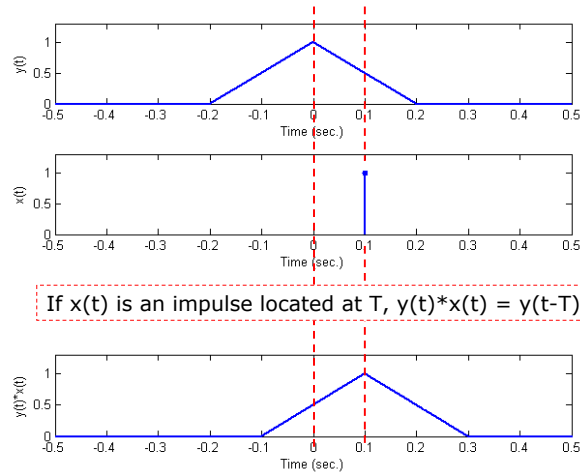
$$x(t) = \begin{cases} 0 & ; -\infty < t < 0, 0.5 \leq t < \infty \\ 1 & ; 0 \leq t < 0.5 \end{cases}$$

$$y(t) = \begin{cases} 0 & ; -\infty < t < 0.2, 0.6 \leq t < \infty \\ 1 & ; 0.2 \leq t < 0.6 \end{cases}$$

$$x(t) * y(t) = \begin{cases} 0 & ; -\infty < t < 0.2 \\ t - 0.2 & ; 0.2 \leq t < 0.6 \\ 0.4 & ; 0.6 \leq t < 0.7 \\ -t + 1.1 & ; 0.7 \leq t < 1.1 \\ 0 & ; 1.1 \leq t < \infty \end{cases}$$



Convolution



Fourier Transform

$$x(t) \Leftrightarrow X(f)$$

Inverse
Fourier
Transform

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df$$

Fourier
Transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$



Magnitude and Phase

Fourier transform can be a complex number.

$$x(t) \Leftrightarrow X(f) = a + jb$$

$$|X(f)| = \sqrt{a^2 + b^2}$$

$$\angle X(f) = \arctan\left(\frac{b}{a}\right)$$



Fourier Transform of an Impulse

$$x(t) = \delta(t - t_0)$$

$$X(f) = \int_{-\infty}^{\infty} \delta(t - t_0) e^{-j2\pi ft} dt = e^{-j2\pi ft_0}$$

$$\therefore \delta(t - t_0) \Leftrightarrow e^{-j2\pi f t_0}$$



Magnitude and Phase of Complex Expo.

Useful Relationship

$$e^{jx} = \cos(x) + j \sin(x)$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{j2}$$

$$|e^{jx}| = |\cos(x) + j \sin(x)|$$

$$= \sqrt{\cos(x)^2 + \sin(x)^2}$$

$$= 1$$

$$\angle e^{jx} = \angle(\cos(x) + j \sin(x))$$

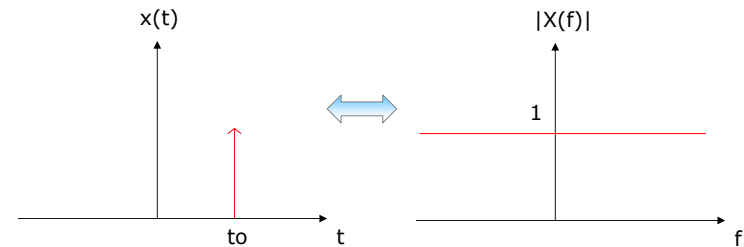
$$= \arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

$$= x$$



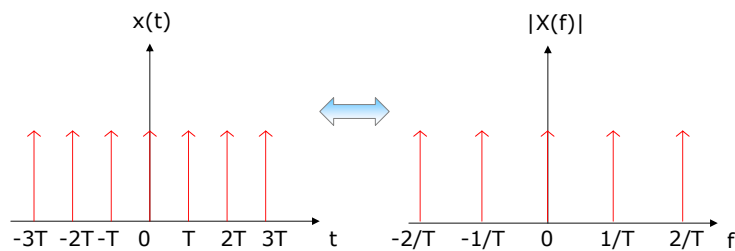
Fourier Transform of an Impulse

$$\delta(t - t_0) \Leftrightarrow e^{-j2\pi f t_0}$$



Fourier Transform of Impulse Train

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \Leftrightarrow \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$



Fourier Transform of Pure Sine

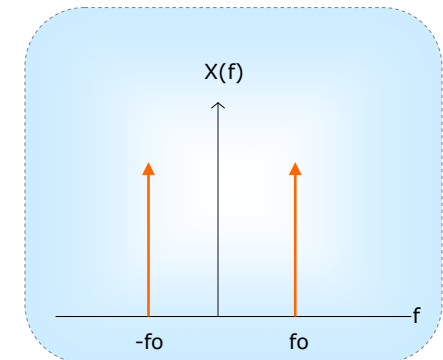
$$x(t) = \cos(2\pi f_0 t)$$

$$X(f) = \int_{-\infty}^{\infty} \cos(2\pi f_0 t) e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} \frac{e^{-j2\pi(f-f_0)t} + e^{-j2\pi(f+f_0)t}}{2} dt$$

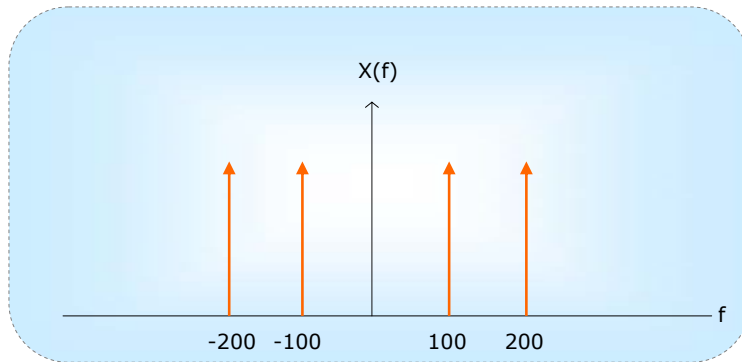
$$= \frac{1}{2} (\delta(f - f_0) + \delta(f + f_0))$$





Frequency Components

$$x(t) = \cos(2\pi(100)t) + \cos(2\pi(200)t)$$



Fourier Series

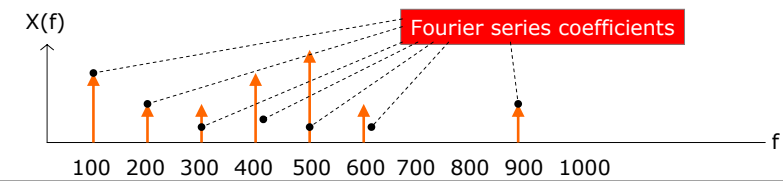
- Linear combination of sinusoidal signals with frequency:

F_0

$nF_0; n = 2, 3, 4, \dots$

results in periodic signals.

- Their Fourier transforms are discrete.



Some Important Properties

$$x(t) * y(t)$$



$$X(f)Y(f)$$

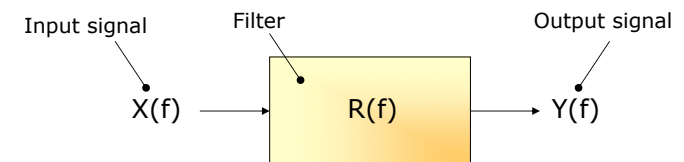
$$x(t)y(t)$$



$$X(f) * Y(f)$$



Linear System



Impulse Response → response of the system when the input is an impulse.

Transfer Function (R(f))

$$\rightarrow R(f) = Y(f)/X(f)$$

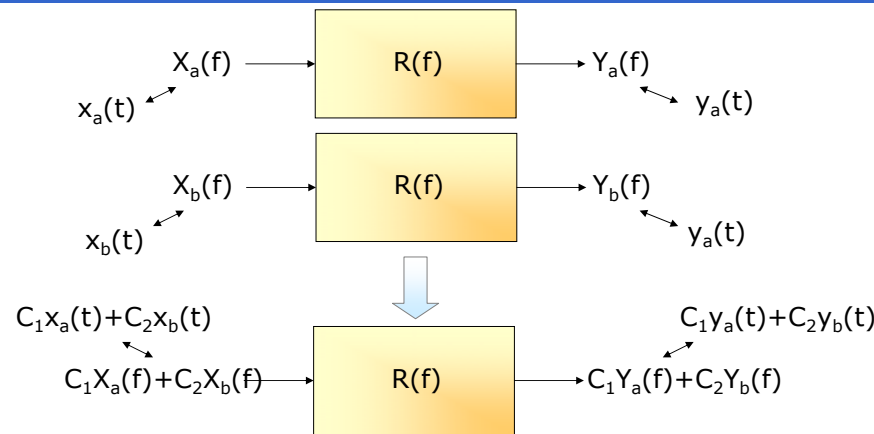
→ R(f) is the Fourier transform of the impulse response of the filter.

$$Y(f) = X(f)R(f)$$



$$y(t) = x(t) * r(t)$$

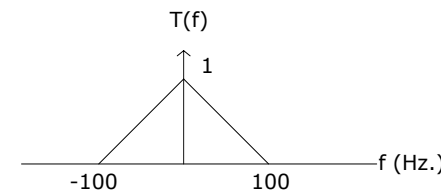
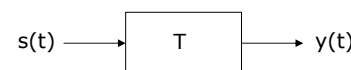
Linear System



Example

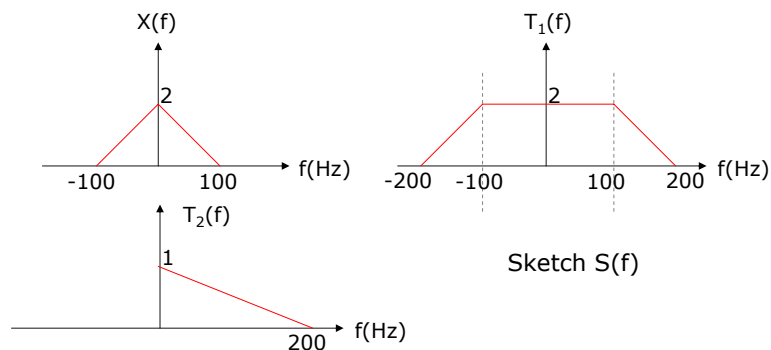
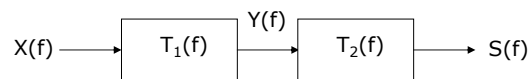
- Find the Fourier transform of:
 $s(t) = 3 \cos(2\pi(50)t) - \sin(2\pi(100)t)$

- For a linear system below:



Find $y(t)$.

Example



Window Effect

$$x(t) = \cos(2\pi f_0 t) \longrightarrow x(t) \text{ has infinite length.}$$

But in real life, we can never have a signal of infinite length.

A portion of sine wave with finite length can be considered as a windowed version of the infinite length sine wave.

E.g.: A sine wave from t_1 to t_2 , $x_w(t)$ can be written as:

$$x_w(t) = \cos(2\pi f_0 t) w(t)$$

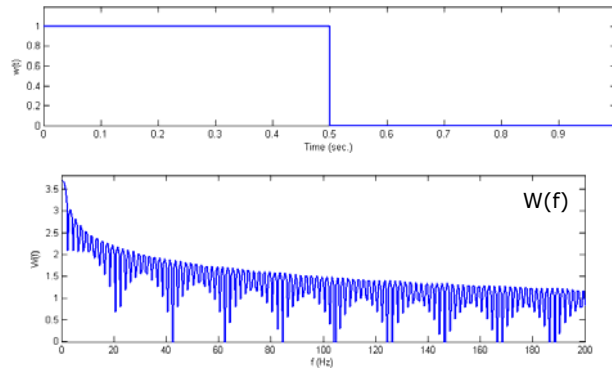
Where:

$$w(t) = \begin{cases} 1 & ; t_1 \leq t < t_2 \\ 0 & ; \text{otherwise} \end{cases}$$



Square Window

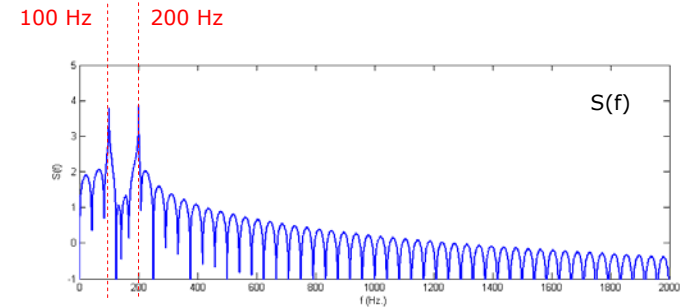
$$w(t) = \begin{cases} 1 & ;0 \leq t < 0.5 \text{ sec.} \\ 0 & ;\text{otherwise} \end{cases}$$



Window Effect

$$s(t) = (\sin(2\pi(100)t) + \sin(2\pi(200)t))w(t)$$

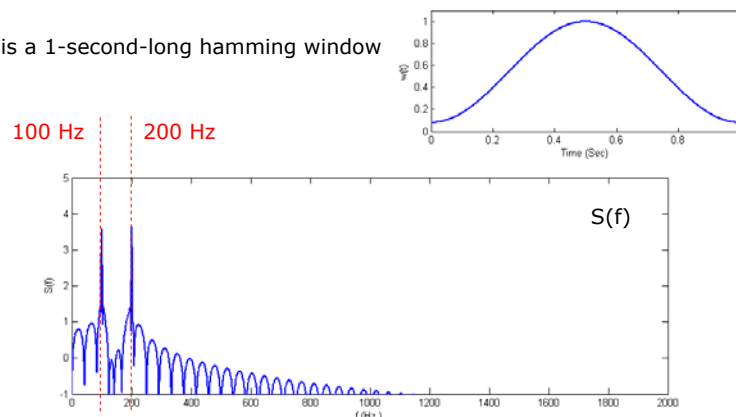
w(t) is a 1-second-long square window



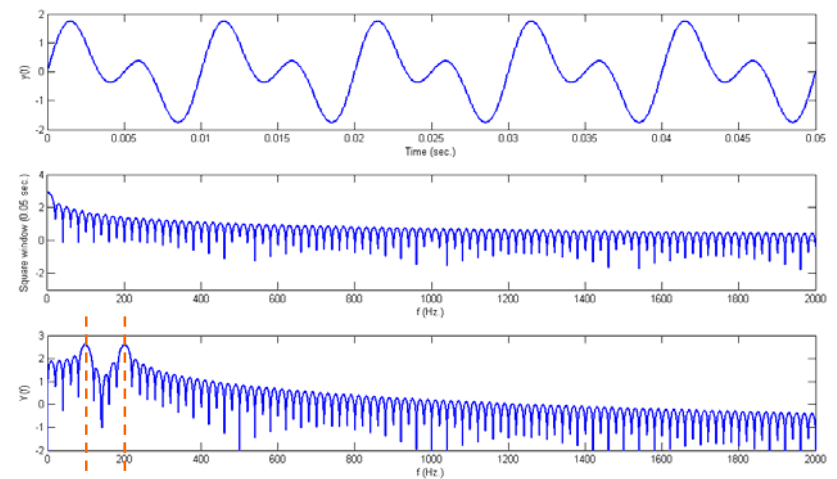
Window Effect

$$s(t) = (\sin(2\pi(100)t) + \sin(2\pi(200)t))w(t)$$

w(t) is a 1-second-long hamming window

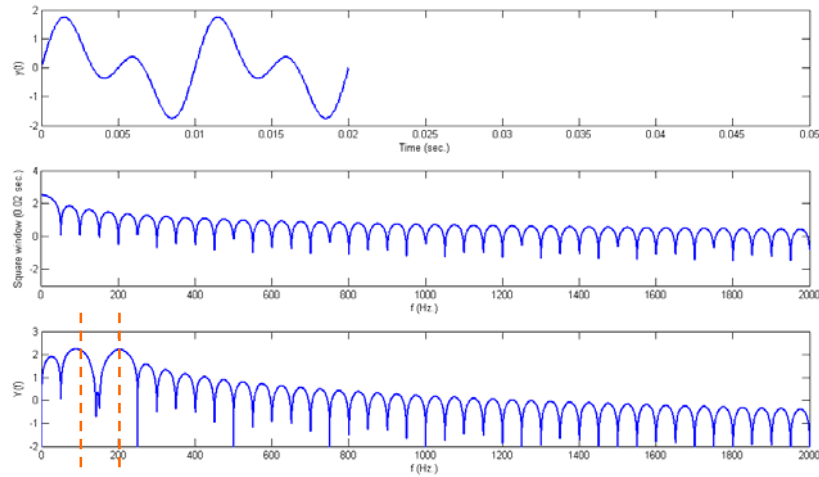


Window Length

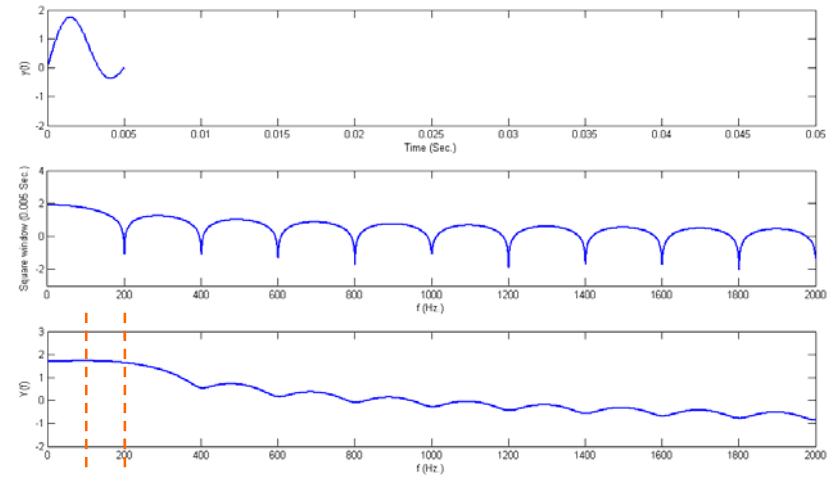




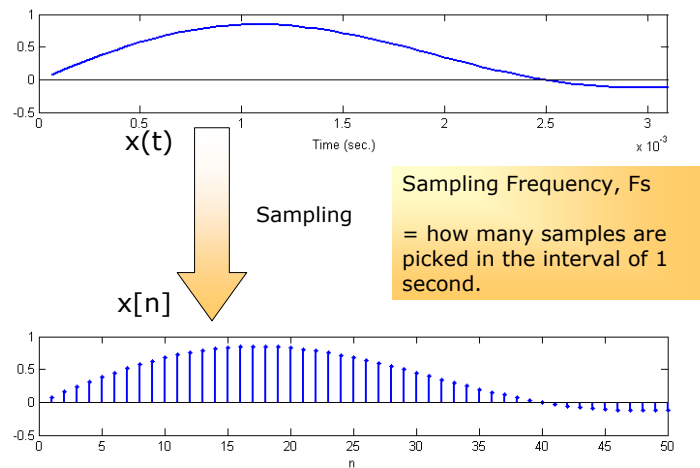
Window Length



Window Length



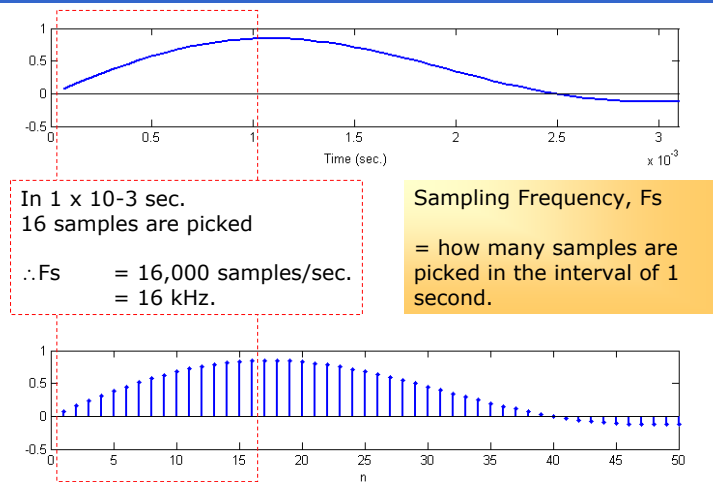
Discrete-time Signal



Sampling Frequency, F_s
= how many samples are picked in the interval of 1 second.



Sampling

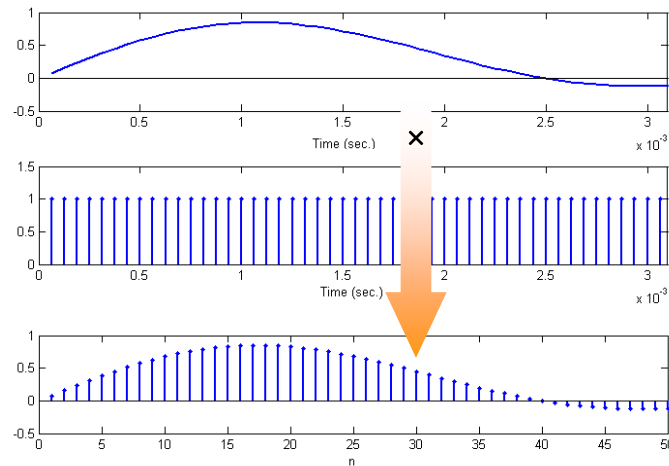


In 1×10^{-3} sec.
16 samples are picked
 $\therefore F_s = 16,000$ samples/sec.
 $= 16$ kHz.

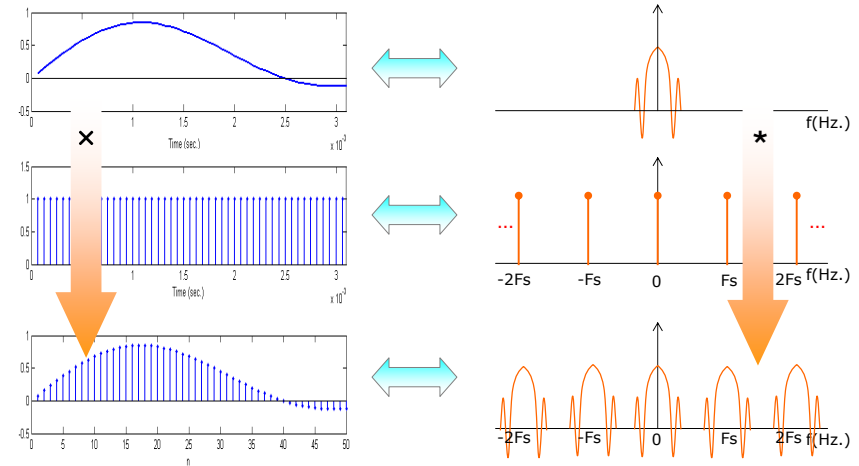
Sampling Frequency, F_s
= how many samples are picked in the interval of 1 second.



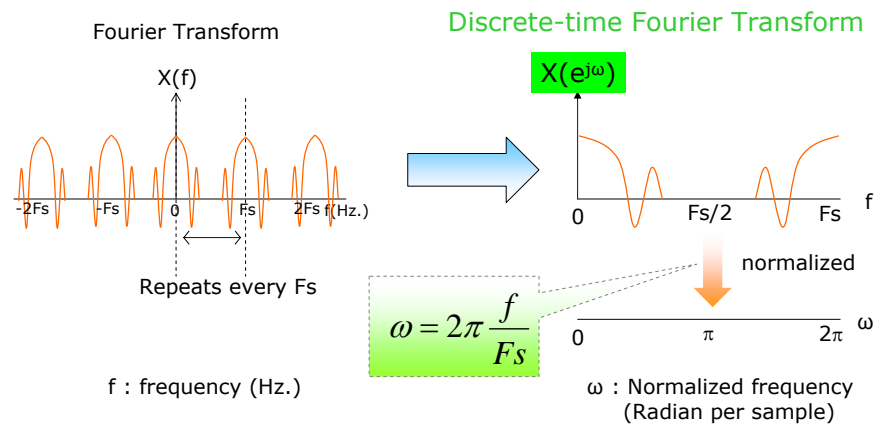
Sampling



Sampling



Normalized Frequency



Discrete-time Fourier Transform

$$x[n] \Leftrightarrow X(e^{j\omega})$$

Discrete-time Fourier Transform

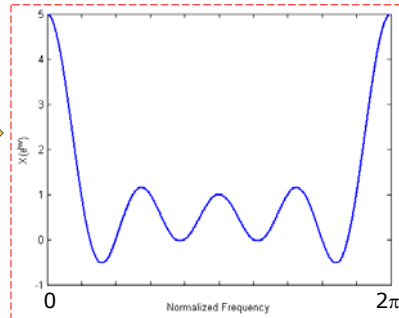
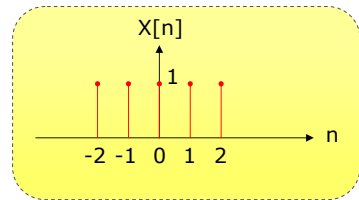
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Discrete-time Fourier Transform

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega$$



Example



$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$= x[-2]e^{-j\omega(-2)} + x[-1]e^{-j\omega(-1)} + x[0]e^{-j\omega(0)} + x[1]e^{-j\omega(1)} + x[2]e^{-j\omega(2)}$$

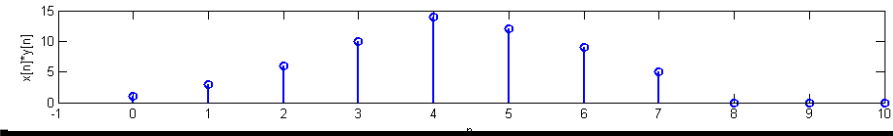
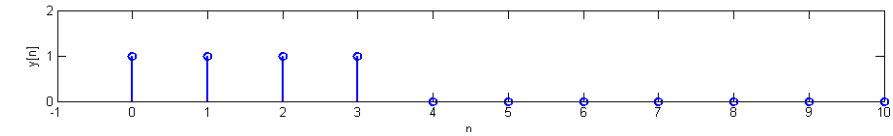
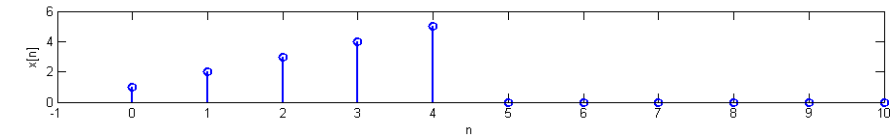
$$= e^{j\omega(2)} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j\omega(2)}$$

$$= 1 + 2\cos(\omega) + 2\cos(2\omega)$$

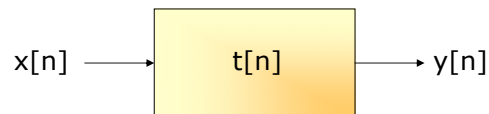


Discrete-time Convolution

$$x[n] * y[n] = \sum_{m=-\infty}^{\infty} x[m]y[n-m]$$



Discrete-time Filter



$$Y(e^{j\omega}) = X(e^{j\omega})T(e^{j\omega})$$

$$y[n] = x[n] * t[n]$$



Family of Fourier Transform

- "Discreteness" in one domain implies "Periodicity" in the other domain.
- "Continuity" in one domain implies "Aperiodicity" in the other domain.

Transform	Time	Freq.
Cont. Fourier Trans.	Cont. & aperiodic	Cont. & aperiodic
Fourier Series	Cont. & periodic	Disc. & aperiodic
DTFT	Disc. & aperiodic	Cont. & periodic
DFT	Disc. & periodic	Disc. & aperiodic



Discrete Fourier Transform

$$x[n] \Leftrightarrow X[k]$$

DFT

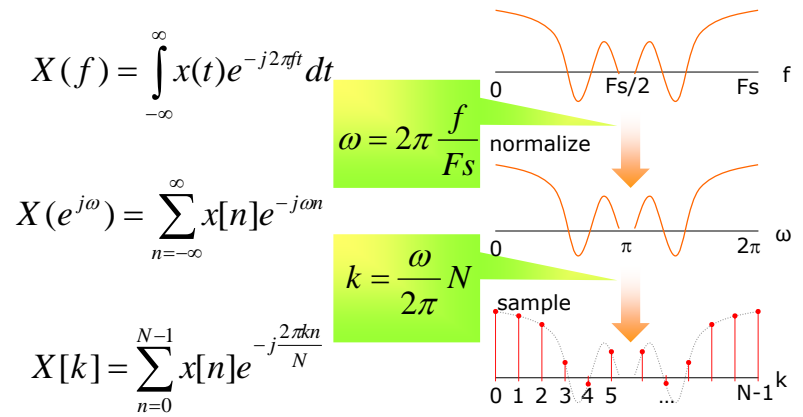
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi kn}{N}} \quad k = 0, 1, \dots, N-1$$

IDFT

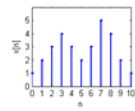
$$X[n] = \frac{1}{N} \sum_{k=0}^{N-1} x[k] e^{j \frac{2\pi kn}{N}} \quad n = 0, 1, \dots, N-1$$



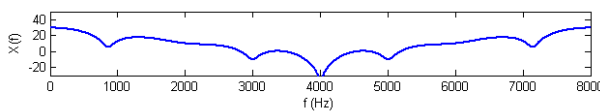
Discrete Fourier Transform



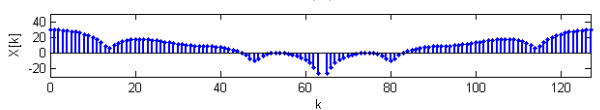
N DFT



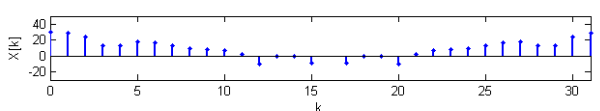
DTFT



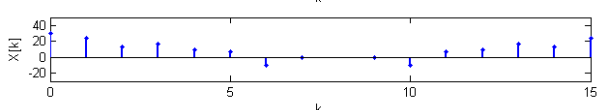
DFT
 N=128



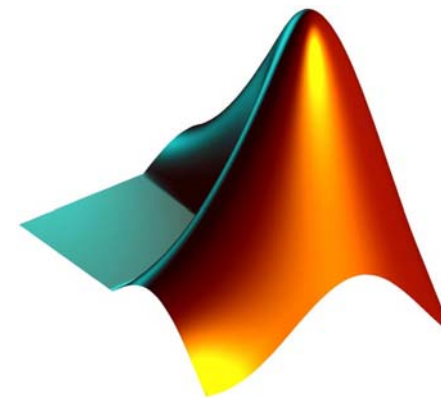
DFT
 N=32



DFT
 N=16



Matlab Demo



- Basic operations
 - Variables
 - Colon operator
 - Matrix manipulation
- Arrays and Cells
- Plotting
- Loading/Saving MS wav
- Help & lookfor
- Control structures