Proof Strategy & Mathematical Induction

- Readings:

  Mathematical Induction:

  Rosen Section_

Mathematical Induction

- A proof by induction that *P(n)* is true for every positive integer *n* consists of 2 steps:

  BASIC STEP: Show that *P(1)* is true.

  INDUCTIVE STEP:
  Show that $P(k) \rightarrow P(k+1)$ is true for every positive integer *k*

- Example :

  Prove that the sum of the first *n* odd positive integers is n^2.

  P(n):

  Basic Step:

  Inductive Step:

- Example:

  Prove that $n < 2^n$ for all positive integers *n*.

  P(n):

  Basic Step:

  Inductive Step:

- Example :

  Prove that n^3-n is divisible by 3 all positive integers n.

  P(n):

  Basic Step:

  Inductive Step:

Mathematical Induction

- Sometimes we want to prove that $P(n)$ is true for $n = b, b+1, b+2, \ldots$ where b is an integer other than 1.

  BASIC STEP: Show that $P(b)$ is true.

  INDUCTIVE STEP:
  Show that $P(k) \rightarrow P(k+1)$ is true for every positive integer k

Example :

Prove that $H_{2^n} \geq 1+\frac{n}{2}$ $\quad H_j = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{j}$

whenever n is a nonnegative integer.

P(n):

Basic Step:

Inductive Step:

Proving Mathematical Induction

- The well-ordering property:

Every nonempty set of nonnegative integers has a least element.

Proving Mathematical Induction

- Show that *P(n)* must be true for all positive integers when *P(1)* and *P(k)→P(k+1)* are true.
- Assume that *P(n)* is not true for at least a positive integer. Then, the set *S* for which *P(n)* is false is nonempty.
- *S* has the least element, called *m*. ($m \neq 1$)
- Since $m-1 < m$, then $m-1 \notin S$ (or *P(m-1)* is true)
- But *P(m-1)→P(m)* is true. So, *P(m)* must be true.
- This contradicts the choice of *m*.

Strong Induction

- A proof by induction that *P(n)* is true for every positive integer *n* consists of 2 steps:
- Use a different induction step.

BASIC STEP: Show that *P(1)* is true.

INDUCTIVE STEP:
Show that $[P(1) \wedge P(2) \wedge \ldots \wedge P(k)] \rightarrow P(k+1)$ is true for every positive integer *k*

- Example:
  Show that if *n* is an integer greater than *1*, then *n* can be written as the product of primes.

  P(n):

  Basic Step:

  Inductive Step:

- Example:
  Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.