Practice Questions for Proofs

1. Prove or disprove that the product of a nonzero rational numbers is rational.
2. Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$
3. The quadratic mean of two real numbers x and y equals $\sqrt{(x^2 + y^2)/2}$. By computing the arithmetic and quadratic means of different pair of positive real numbers, formulate a conjecture about their relative sizes and prove your conjecture.
4. Prove or disprove that you can use dominoes to tile a standard checkerboard with all four corners removed.
5. Use Mathematical Induction to prove that if $A_1, A_2, \ldots, An$ and B are sets, then

$$(A_1 \cap A_2 \cap \ldots \cap An) \cup B = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (An \cup B)$$

6. Suppose that

$$A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

where a and b are real numbers, use Mathematical Induction to show that

$$A^n = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$$

for every positive integer n.