***Part One***

1. Show that these propositions are tautologies.
   a) $(\sim q \wedge (p \to q)) \to \sim p$
   b) $((p \vee q) \wedge \sim p) \to q$

2. If $\forall y \exists x P(x, y)$ is true, does it necessarily follow that $\exists x \forall y P(x, y)$ is true?

3. Prove that if $n$ is an integer and $3n+2$ is even, then $n$ is even using
   a) an indirect proof.
   b) a proof by contradiction.

4. Prove that a square of an integer ends with a 0,1,4,5,6,or 9. (Hint: Let $n = 10k+l$ where l = 0,1,…,9)

5. Prove or disprove that if $a$ and $b$ are rational numbers, then $a^b$ is also rational.

6. Prove that if $x^3$ is irrational, then $x$ is irrational.

7. The successor of the set A is the set $A \cup \{A\}$. Find the successors of the following sets.
   a) $\{1,2,3\}$ b) $\varnothing$
   c) $\{\varnothing\}$ d) $\{\varnothing,\{\varnothing\}\}$

8. Determine whether each of these functions is a bijection from R to R
   a) $f(x) = -3x + 4$
   b) $f(x) = -3x^2 + 7$
   c) $f(x) = (x+1)/(x+2)$
   d) $f(x) = x^5 + 1$

9. Let $f(x) = 2x$. What is
   a) $f(\mathrm{Z})$ b) $f(\mathrm{N})$ c) $f(\mathrm{R})$

10. Determine whether the relation $R$ on the set of all people is reflexive,symmetric, and/or transitive, where $(a,b) \in R$ if and only if
    a) $a$ is taller than $b$.
    b) $a$ and $b$ were born on the same day.
    c) $a$ has the same first name as $b$.
    d) $a$ and $b$ have a common grandparent.

11. Let $R$ be the relation $R = \{(a,b) \mid a < b\}$ on the set of integers. Find
    a) $R^{-1}$ b) $\overline{R}$

12. Suppose that $R_1$ and $R_2$ are equivalence relations on the set S. Determine whether each of these combinations of $R_1$ and $R_2$ must be an equivalence relation.
    a) $R_1 \cup R_2$ b) $R_1 \cap R_2$ c) $R_1 \oplus R_2$

13. What is the congruence class $[4]_m$ when $m$ is
    a) 2? b) 3? c) 8?