Aug 2001 Algorithm

The Intersections for a Set of 2D Segments, and Testing Simple Polygons

by Dan Sunday

Sometimes an application will need to find the set of intersection points for a collection of many line segments. Often these applications involve polygons which are just an ordered set of connected segments. Specific problems that might need an algorithmic solution are:

1. Test if a polygon is simple. That is, determine if any two nonsequential edges of a polygon intersect. This is an important property since many algorithms only work for simple polygons. This test can stop as soon as any intersection is found, and it doesn't have to determine the complete set of intersections.

2. Decompose a polygon into simple pieces. To do this, one needs to know the complete set of intersection points between the edges, and use each of them as a cut-point in the decomposition.

3. Test if two simple polygons or planar graphs intersect. One has to determine the intersections of one object's edges with those of the other. Again, as soon as any intersection is found, the test can stop.

4. Compute the intersection (or union, or difference) of two simple polygons or planar graphs. To do this, one must determine all intersection points, and use them as new vertices to construct the intersection.

Algorithms solving these problems are used in many application areas such as computer graphics, CAD, circuit design, hidden line elimination, computer vision, and so on.

A Short Survey of Intersection Algorithms

In general, for a set of n segments, there can be up to $O(n^2)$ intersection points, since if every segment intersected every other segment, there would be $(n\text{-}1)+(n\text{-}2)+...+1 = n(n\text{-}1)/2 = O(n^2)$ intersection points. In the worst case, to compute them all would require a $O(n^2)$ algorithm. The "brute force" algorithm would simply consider all $O(n^2)$ pairs of line segments, test each pair for intersection, and record the ones it finds. This is a lot of computing. However, when there are only a few intersection points, or only one such point needs to be detected (or not), then are faster algorithms.

In fact, these problems can be solved by "output-sensitive" algorithms whose efficiency depends on both the input and the output sizes. Here the input is a set Ω of n segments, and the output is the set Λ of k computed intersections. An early algorithm [Shamos & Hoey, 1976] showed how to detect if at least one intersection exists in $O(n \log n)$ time and $O(n)$ space by "sweeping" over a linear ordering of Ω. Extending their idea, [Bentley & Ottmann, 1979] gave an algorithm to compute all k intersections in $O((n+k) \log n)$ time and $O(n+k)$ space. After more than 20 years, the well-known "***Bentley-Ottmann Algorithm***" is still the most popular one to implement in practice ([Bartuschka, Mehlhorn & Naher, 1997], [de Berg et al, 2000], [Hobby, 1999], [O'Rourke, 1998], [Preparata & Shamos, 1985]) since it is relatively easy to both understand and implement. However, their algorithm did not achieve the theoretical lower bound; and thus, was only the first of many output-sensitive algorithms for solving the segment intersection problem.

A decade later, [Chazelle & Edelsbrunner, 1988 and 1992] discovered an optimal $O(n \log n + k)$ time algorithm. But, their algorithm still needs $O(n+k)$ storage space, and it is difficult to implement. Subsequent work made further improvements, and [Balaban, 1995] found an $O(n \log n + k)$ time and $O(n)$ space deterministic algorithm. There have also been a number of "randomized" algorithms with *expected*

$O(n \log n + k)$ running time. The earliest of these by [Myers, 1985] uses $O(n+k)$ space. However, the later one by [Clarkson & Shor, 1989] uses only $O(n)$ space.

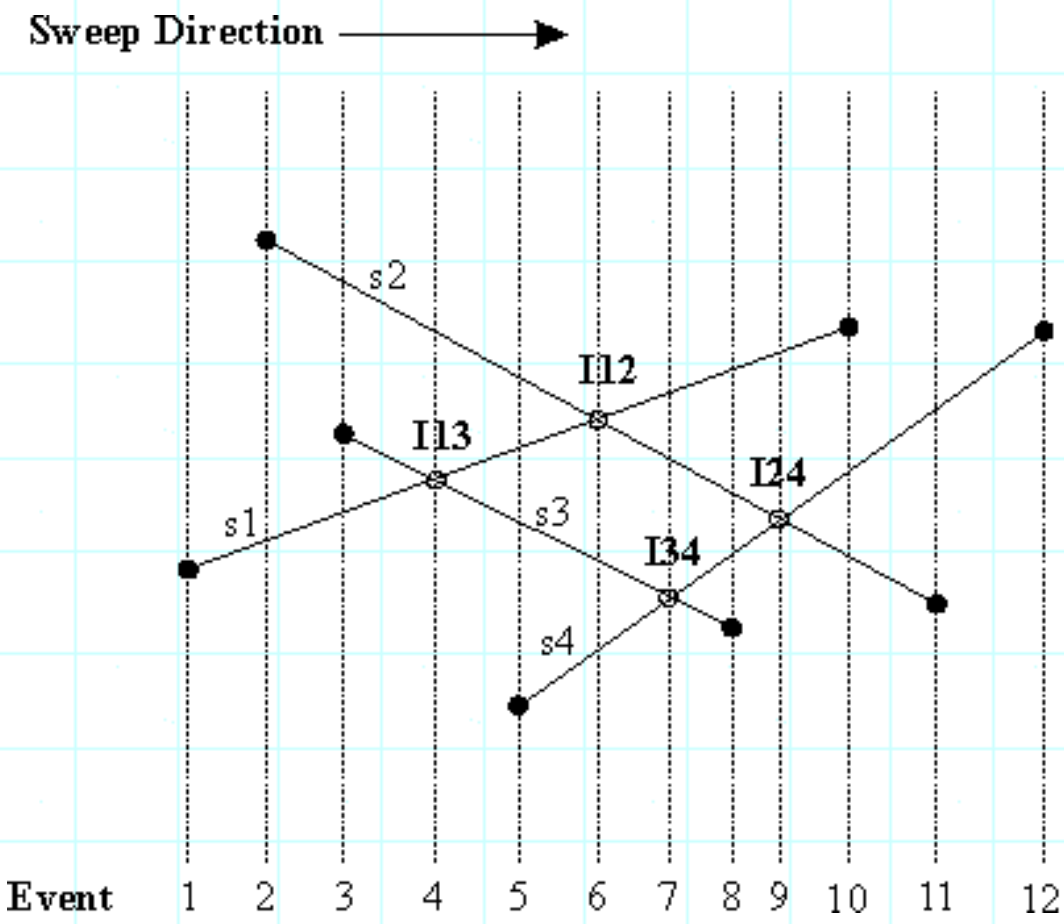
Additionally, improved algorithms have been found for the more restrictive "red-blue intersection" problem. Here there are two separate sets of segments, the "red" set Ω_1 and the "blue" set Ω_2. One wants to find intersections between the sets, but not within the same set; that is, red-blue intersections, but not red-red or blue-blue ones. A simple deterministic $O(n \log n + k)$ time and $O(n)$ space "trapezoid sweep" algorithm was developed by [Chan, 1994] based on earlier work of [Mairson & Stolfi, 1988]. These algorithms can be used to perform boolean operations, like intersections or unions, between two different polygons or planar subdivision graphs.

Nevertheless, the Shamos-Hoey and Bentley-Ottmann algorithms remain the landmarks of the field. Note, however, that when k is large of order $O(n^2)$, the Bentley-Ottmann algorithm takes $O(n^2 \log n)$ time which is worse than the $O(n^2)$ brute-force algorithm! Also, the more complicated optimal algorithms are $O(n^2)$ which is the same as the simple brute-force one. So, one might as well use the easy-to-implement brute-force algorithm when k is expected to be much larger than $O(n)$. But, when k is expected to be less than or equal to $O(n)$, Bentley-Ottmann is the simplest expected $O(n \log n)$ time and $O(n)$ space algorithm.

The Bentley-Ottmann Algorithm

The input for the Bentley-Ottmann algorithm is a collection $\Omega = \{\mathbf{L}_i\}$ of line segments $\mathbf{L}_i$, and its output will be a set $\Lambda = \{I_j\}$ of intersection points. This algorithm is referred to as a "***sweep line algorithm***" because it's operation can be visualized as having another "sweep line" **SL** sweeping over the collection Ω and collecting information as it passes over the individual segments $\mathbf{L}_i$. The information collected is basically an ordered list of all segments in Ω that are currently intersected by **SL**. The data structure maintaining this information is often also called the "sweep line". It also detects and outputs intersections as it discovers them. The process by which it discovers intersections is the heart of the algorithm and its efficiency.

To implement the sweep logic, we must first linearly order the segments of Ω to determine the sequence in which **SL** encounters them. Actually, we need to order the endpoints $\{E_{i0}, E_{i1}\}$ of all segments $\mathbf{L}_i$ so we can detect when **SL** starts and stops to intersecting each $\mathbf{L}_i$. Traditionally, endpoints are ordered by increasing x and then increasing y-coordinate values, but any linear order will do (some authors prefer a decreasing y and then increasing x value). With the traditional ordering, the sweep line is vertical and moves from left to right as it encounters each segment, as shown in the diagram:



At any point in the algorithm, the sweep line **SL** intersects those segments with one endpoint to the left of it and the other endpoint to the right. The **SL** data structure keeps track of these segments by: (1) adding a segment when its left endpoint is encountered, and (2) deleting a segment when its right endpoint is encountered. The **SL** also maintains the segments in a list ordered by an "above-below" relation. So, to add or delete a segment, its position in the list must be determined, and this can be done by a worst-case O (log n) binary search of the current **SL** segments. But, besides adding or deleting segments, there is another event that changes the structure of the **SL**; namely, whenever two current segments intersect, then their positions in the ordered list are swapped. Given the two segments, which must be neighbors in the list, this swap is an O(log n) operation.

To organize all this, the algorithm has an ordered "***event queue***" ξ whose elements cause a change in the **SL** segment list. Initially, ξ is set to the sweep-ordered list of all segment endpoints. But as intersections between segments are found, then they are also added to ξ in the same sweep-order as used for the endpoints One must test, though, to avoid inserting duplicate intersections onto the event queue. The example in the above diagram shows how this can happen. At event 2, segments s1 and s2 cause intersection I12 to be computed and put on the queue. Then, at event 3, segment s3 comes between and separates s1 and s2. Next, at event 4, s1 and s3 swap places on the sweep line, and s1 is brought next to s2 again causing I12 to be computed again. But, there can only be one event for each intersection, and I12 cannot be put on the queue again. So, when an intersection is being put on the queue, we must find its potential x-sorted location in the queue, and check that it is not already there. Since there is at most one intersect point for any two segments, labeling an intersection with identifiers for the segments is sufficient to uniquely identify it. As a result of all this, the maximum size of the event queue = $2n + k \leq 2n + n^2$,

and an insertion into it can be done with at worst an O(log $(2n + n^2)$) = O(log n) binary search.

But, what does all this have to do with efficiently finding the complete set of segment intersections? Well, as segments are sequentially updated on the **SL** list, their possible intersection with other eligible segments is determined. When a valid intersection is found, then it is inserted into the event queue. Further, when an intersection-event on ξ is processed during the sweep, then it causes a re-ordering of the **SL** list, and is also added to the output list Λ. In the end, when all events have been processed, Λ will contain the complete set of all intersections.

However, there is one critical detail, the heart of the algorithm, that we still need to describe; namely, how does one compute a valid intersection? Clearly, two segments can only intersect if they occur simultaneously on the sweep-line at some time. But this by itself is not enough to make the algorithm efficient. The important observation is that two intersecting segments must be immediate above-below neighbors on the sweep-line. Thus, there are just a few restricted cases for which possible intersections must be computed:

1. When a segment is added to the **SL**, determine if it intersects with its above and below neighbors.

2. When a segment is deleted from the **SL**, its previous above and below neighbors are brought together as new neighbors. So, their possible intersection needs to be determined.

3. At an intersection event, two segments switch positions in the **SL**, and their intersection with their new neighbors must be determined.

This means that for the processing of any one event (endpoint or intersection) of ξ, there are at most 2 intersection determinations that need to be made. One detail remains, namely the time needed to add, find, swap, and remove segments from the **SL** structure. To do this, implement **SL** as a balanced binary tree (such as an AVL, a 2-3, or a red-black tree) which guarantees that these operations take at most O(log n) time since n is the maximum size of **SL**. Thus, each of the $(2n+k)$ events has at worst O(log n) processing to do. Adding everything up, the efficiency of the algorithm = O(initial-sort) + O(event-processing) = O(n log n) + O($(2n+k)$ log n) = O($(n+k)$ log n).

Pseudo-Code: Bentley-Ottmann Algorithm

Putting all of this together, the top-level logic for an implementation of the Bentley-Ottmann algorithm is given by the following pseudo-code:

```
    Initialize event queue ξ = all segment endpoints;
    Sort ξ by increasing x and y;
    Initialize sweep line SL to be empty;
    Initialize output intersection list Λ to be empty;

    While (ξ is nonempty) {
        Let E = the next event from ξ;
        If (E is a left endpoint) {
            Let segE = E's segment;
            Add segE to SL;
            Let segA = the segment above segE in SL;
            Let segB = the segment below segE in SL;
            If (I = Intersect( segE with segA) exists)
                Insert I into ξ;
            If (I = Intersect( segE with segB) exists)
                Insert I into ξ;
        }
        Else If (E is a right endpoint) {
            Let segE = E's segment;
            Let segA = the segment above segE in SL;
            Let segB = the segment below segE in SL;
            Remove segE from SL;
            If (I = Intersect( segA with segB) exists)
                If (I is not in ξ already) Insert I into ξ;
        }
        Else {  // E is an intersection event
            Add E to the output list Λ;
            Let segE1 above segE2 be E's intersecting segments in SL;
            Swap their positions so that segE2 is now above segE1;
            Let segA = the segment above segE2 in SL;
            Let segB = the segment below segE1 in SL;
            If (I = Intersect(segE2 with segA) exists)
                If (I is not in ξ already) Insert I into ξ;
            If (I = Intersect(segE1 with segB) exists)
                If (I is not in ξ already) Insert I into ξ;
        }
        remove E from ξ;
    }
    return Λ;
}
```

This routine outputs the complete list of all intersection points.

Pseudo-Code: Shamos-Hoey Algorithm

If one only wants to know if an intersection exists, then as soon as any intersection is detected, the routine can terminate immediately. This results in a simplified algorithm. Intersections don't ever have to be put on the event queue, and so its size is just 2*n* for the endpoints of every segment. Further, code for processing this non-existent event can be removed. Also, the event (priority) queue can be implemented as a simple ordered array since it never changes. Additionally, no output list needs to be built since the algorithm terminates as soon as any intersection is found. Thus, this algorithm needs only O(*n*) space and runs in O(*n* log *n*) time. This is the original algorithm of [Shamos & Hoey, 1976]. The pseudo-code for this simplified routine is:

```
    Initialize event queue ξ = all segment endpoints;
    Sort ξ by increasing x and y;
    Initialize sweep line SL to be empty;

    While (ξ is nonempty) {
        Let E = the next event from ξ;
        If (E is a left endpoint) {
            Let segE = E's segment;
            Add segE to SL;
            Let segA = the segment above segE in SL;
            Let segB = the segment below segE in SL;
            If (I = Intersect( segE with segA) exists)
                return TRUE;   // an Intersect Exists
            If (I = Intersect( segE with segB) exists)
                return TRUE;   // an Intersect Exists
        }
        Else {  // E is a right endpoint
            Let segE = E's segment;
            Let segA = the segment above segE in SL;
            Let segB = the segment below segE in SL;
            Delete segE from SL;
            If (I = Intersect( segA with segB) exists)
                return TRUE;   // an Intersect Exists
        }
        remove E from ξ;
    }
    return FALSE;      // No Intersections
}
```

Applications

Simple Polygons

The Shamos-Hoey algorithm can be used to ***test if a polygon is simple or not***. We give a C++ implementation simple_Polygon() for this algorithm below. Note that the shared endpoint between sequential edges does not count as a non-simple intersection point, and the intersection test routine must check for that.

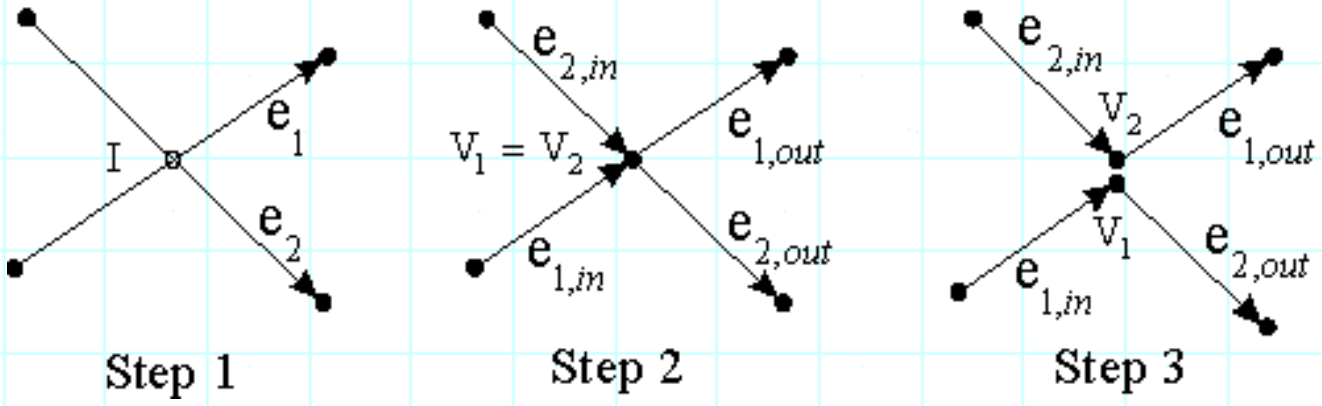
The Bentley-Ottmann algorithm can be used to ***decompose a non-simple polygon into simple pieces***. To do this, all intersection points are needed. One approach for a simple decomposition algorithm is to perform the following operations:

1. Compute all the intersection points using the Bentley-Ottmann algorithm

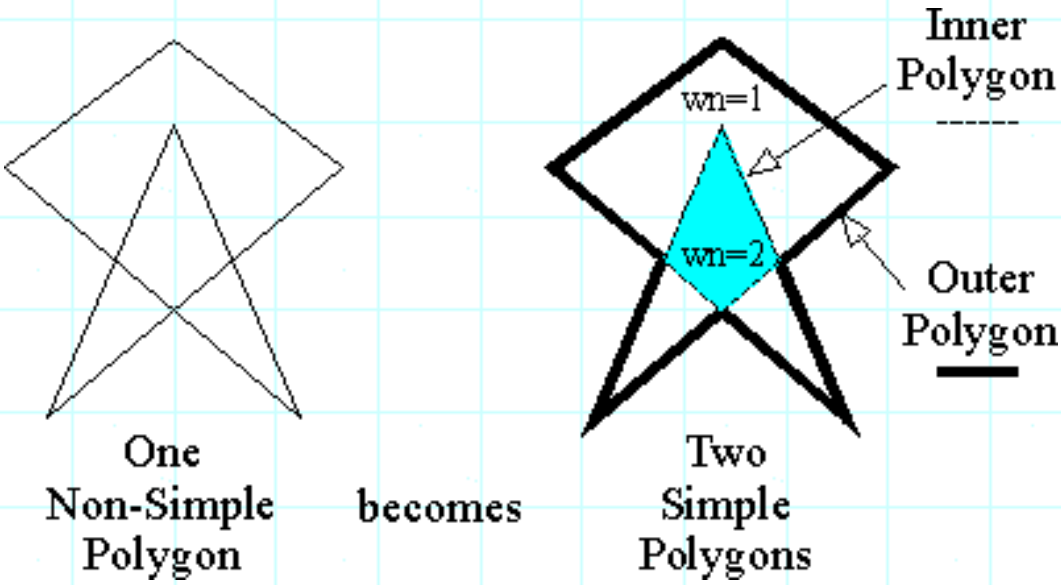
2. For each intersection, add 2 new vertices V_1 and V_2 (one on each edge $\mathbf{e}_1$ and $\mathbf{e}_2$), and split each edge $\mathbf{e}_i$ into two new edges $\mathbf{e}_{i,in}$ and $\mathbf{e}_{i,out}$ joined at the new vertex V_i.

3. Do surgery at these new vertices to remove the crossover. This is done by
   a) attaching $\mathbf{e}_{1,in}$ to $\mathbf{e}_{2,out}$ at V_1, and
   b) attaching $\mathbf{e}_{2,in}$ to $\mathbf{e}_{1,out}$ at V_2
   as shown in the following diagram.

4. After doing this at all intersections, then the remaining connected edge sets are the simple polygons decomposing the original non-simple one.



Note that the resulting simple polygons may not be disjoint since one could be contained inside another. In fact, the decomposition inclusion hierarchy is based on the inclusion winding number of each simple polygon in the original non-simple one (see the March 2001 Algorithm on Winding Number Inclusion). For example:



Polygon Set Operations

The Bentley-Ottmann algorithm can be used to speed up computing the intersection, union, or difference of two general non-convex simple polygons. Of course, before using any complicated algorithm to perform these operations, one should first test the bounding boxes or spheres of the polygons for overlap (see the July 2001 Algorithm on Bounding Containers). If the bounding containers are disjoint, then so are the two polygons, and the set operations become trivial.

However, when two polygons overlap, the sweep line strategy of the Bentley-Ottmann algorithm can be adapted to perform a set operation on any two simple polygons. For further details see [O'Rourke, 1998, 266-269]. If the two polygons are known to be simple, then one just needs intersections for segments from different polygons. So, this is a red-blue intersection problem, and could be solved by an efficient red-blue algorithm.

Planar Subdivisions

The Bentley-Ottmann algorithm can be used to efficiently compute the overlay of two planar subdivisions. For details, see [de Berg et al, 2000, 33-39]. A planar subdivision is a planar graph with straight line segments for edges, and it divides the plane into a finite number of regions. For example, boundary lines divide a country into states. When two such planar graphs are overlaid (or superimposed), they their combined graph defines a refined subdivision of each one. To compute this refinement, one needs to calculate all intersections between the line segments in both graphs. For a segment in one graph, we only need the intersections with segments in the other graph, and so this is a red-blue intersection problem which could be solved with an efficient red-blue algorithm.

Implementations

Here are some sample "C++" implementations of these algorithms.

```
// Copyright 2001, softSurfer (www.softsurfer.com)
// This code may be freely used and modified for any purpose
// providing that this copyright notice is included with it.
// SoftSurfer makes no warranty for this code, and cannot be held
// liable for any real or imagined damage resulting from its use.
// Users of this code must verify correctness for their application.


// Assume that classes are already given for the objects:
//    Point with 2D coordinates {float x, y;}
//    Polygon with n vertices {int n; Point *V;} with V[n]=V[0]
//    Tnode is a node element structure for a BBT
//    BBT is a class for a Balanced Binary Tree
//        such as an AVL, a 2-3, or a red-black tree
//        with methods given by the placeholder code:

typedef struct _BBTnode Tnode;
struct _BBTnode {
       void* val;
       // plus node mgmt info ...
};

class BBT {
       Tnode   *root;
public:
              BBT() {root = (Tnode*)0;}   // constructor
              ~BBT() {freetree();}        // destructor

       Tnode*  insert( void* ){};      // insert data into the tree
       Tnode*  find( void* ){};        // find data from the tree
       Tnode*  next( Tnode* ){};       // get next tree node
       Tnode*  prev( Tnode* ){};       // get previous tree node
       void    remove( Tnode* ){};     // remove node from the tree
       void    freetree(){};           // free all tree data structs
};
// NOTE:
// Code for these methods must be provided for the algorithm to work.
// We have not provided it since binary tree algorithms are well-known
// and code is widely available.  Further, we want to reduce the
```

```
clutter
// accompanying the essential sweep line algorithm.
//===================================================================



#define FALSE   0
#define TRUE    1
#define LEFT    0
#define RIGHT   1

extern void
qsort(void*, unsigned, unsigned, int(*)(const void*,const void*));

// xyorder(): determines the xy lexicographical order of two points
//      returns: (+1) if p1 > p2; (-1) if p1 < p2; and 0 if equal
int xyorder( Point* p1, Point* p2 )
{
    // test the x-coord first
    if (p1->x > p2->x) return 1;
    if (p1->x < p2->x) return (-1);
    // and test the y-coord second
    if (p1->y > p2->y) return 1;
    if (p1->y < p2->y) return (-1);
    // when you exclude all other possibilities, what remains is...
    return 0;  // they are the same point
}

// isLeft(): tests if point P2 is Left|On|Right of the line P0 to P1.
//      returns: >0 for left, 0 for on, and <0 for right of the line.
//      (see the January 2001 Algorithm on Area of Triangles)
inline float
isLeft( Point P0, Point P1, Point P2 )
{
    return (P1.x - P0.x)*(P2.y - P0.y) - (P2.x - P0.x)*(P1.y - P0.y);
}
//===================================================================



// EventQueue Class

// Event element data struct
typedef struct _event Event;
struct _event {
    int      edge;        // polygon edge i is V[i] to V[i+1]
```

```
    int      type;         // event type: LEFT or RIGHT vertex
    Point*   eV;           // event vertex
};

int E_compare( const void* v1, const void* v2 ) // qsort compare two
events
{
    Event**    pe1 = (Event**)v1;
    Event**    pe2 = (Event**)v2;

    return xyorder( (*pe1)->eV, (*pe2)->eV );
}

// the EventQueue is a presorted array (no insertions needed)
class EventQueue {
    int      ne;              // total number of events in array
    int      ix;              // index of next event on queue
    Event*   Edata;           // array of all events
    Event**  Eq;              // sorted list of event pointers
public:
             EventQueue(Polygon P);     // constructor
             ~EventQueue(void)          // destructor
                 { delete Eq; delete Edata;}

    Event*   next();                    // next event on queue
};

// EventQueue Routines
EventQueue::EventQueue( Polygon P )
{
    ix = 0;
    ne = 2 * P.n;          // 2 vertex events for each edge
    Edata = (Event*)new Event[ne];
    Eq = (Event**)new (Event*)[ne];
    for (int i=0; i < ne; i++)          // init Eq array pointers
        Eq[i] = &Edata[i];

    // Initialize event queue with edge segment endpoints
    for (int i=0; i < P.n; i++) {       // init data for edge i
        Eq[2*i]->edge = i;
        Eq[2*i+1]->edge = i;
        Eq[2*i]->eV   = &(P.V[i]);
        Eq[2*i+1]->eV = &(P.V[i+1]);
        if (xyorder( &P.V[i], &P.V[i+1]) < 0) { // determine type
            Eq[2*i]->type   = LEFT;
```

```
            Eq[2*i+1]->type = RIGHT;
        }
        else {
            Eq[2*i]->type   = RIGHT;
            Eq[2*i+1]->type = LEFT;
        }
    }
    // Sort Eq[] by increasing x and y
    qsort( Eq, ne, sizeof(Event*), E_compare );
}

Event* EventQueue::next()
{
    if (ix >= ne)
        return (Event*)0;
    else
        return Eq[ix++];
}
//==================================================================



// SweepLine Class

// SweepLine segment data struct
typedef struct _SL_segment SLseg;
struct _SL_segment {
    int      edge;         // polygon edge i is V[i] to V[i+1]
    Point    lP;           // leftmost vertex point
    Point    rP;           // rightmost vertex point
    SLseg*   above;        // segment above this one
    SLseg*   below;        // segment below this one
};

// the Sweep Line itself
class SweepLine {
    int      nv;           // number of vertices in polygon
    Polygon* Pn;           // initial Polygon
    BBT      Tree;         // balanced binary tree
public:
            SweepLine(Polygon P)           // constructor
                { nv = P.n; Pn = &P; }
            ~SweepLine(void)               // destructor
                { Tree.freetree();}
```

```
    SLseg*   add( Event* );
    SLseg*   find( Event* );
    int      intersect( SLseg*, SLseg* );
    void     remove( SLseg* );
};

SLseg* SweepLine::add( Event* E )
{
    // fill in SLseg element data
    SLseg* s = new SLseg;
    s->edge  = E->edge;

    // if it is being added, then it must be a LEFT edge event
    // but need to determine which endpoint is the left one
    Point* v1 = &(Pn->V[s->edge]);
    Point* v2 = &(Pn->V[s->edge+1]);
    if (xyorder( v1, v2) < 0) { // determine which is leftmost
        s->lP = *v1;
        s->rP = *v2;
    }
    else {
        s->rP = *v1;
        s->lP = *v2;
    }
    s->above = (SLseg*)0;
    s->below = (SLseg*)0;

    // add a node to the balanced binary tree
    Tnode* nd = Tree.insert(s);
    Tnode* nx = Tree.next(nd);
    Tnode* np = Tree.prev(nd);
    if (nx != (Tnode*)0) {
        s->above = (SLseg*)nx->val;
        s->above->below = s;
    }
    if (np != (Tnode*)0) {
        s->below = (SLseg*)np->val;
        s->below->above = s;
    }
    return s;
}

SLseg* SweepLine::find( Event* E )
{
```

```
    // need a segment to find it in the tree
    SLseg* s = new SLseg;
    s->edge  = E->edge;
    s->above = (SLseg*)0;
    s->below = (SLseg*)0;

    Tnode* nd = Tree.find(s);
    delete s;
    if (nd == (Tnode*)0)
        return (SLseg*)0;

    return (SLseg*)nd->val;
}

void SweepLine::remove( SLseg* s )
{
    // remove the node from the balanced binary tree
    Tnode* nd = Tree.find(s);
    if (nd == (Tnode*)0)
        return;      // not there !

    // get the above and below segments pointing to each other
    Tnode* nx = Tree.next(nd);
    if (nx != (Tnode*)0) {
        SLseg* sx = (SLseg*)(nx->val);
        sx->below = s->below;
    }
    Tnode* np = Tree.prev(nd);
    if (np != (Tnode*)0) {
        SLseg* sp = (SLseg*)(np->val);
        sp->above = s->above;
    }
    Tree.remove(nd);       // now can safely remove it
    delete s;
}

// test intersect of 2 segments and return: 0=none, 1=intersect
int SweepLine::intersect( SLseg* s1, SLseg* s2)
{
    if (s1 == (SLseg*)0 || s2 == (SLseg*)0)
        return FALSE;      // no intersect if either segment doesn't
exist

    // check for consecutive edges in polygon
    int e1 = s1->edge;
```

```
    int e2 = s2->edge;
    if (((e1+1)%nv == e2) || (e1 == (e2+1)%nv))
        return FALSE;      // no non-simple intersect since consecutive

    // test for existence of an intersect point
    float lsign, rsign;
    lsign = isLeft(s1->lP, s1->rP, s2->lP);    // s2 left point sign
    rsign = isLeft(s1->lP, s1->rP, s2->rP);    // s2 right point sign
    if (lsign * rsign > 0) // s2 endpoints have same sign relative to
s1
        return FALSE;      // => on same side => no intersect is
possible
    lsign = isLeft(s2->lP, s2->rP, s1->lP);    // s1 left point sign
    rsign = isLeft(s2->lP, s2->rP, s1->rP);    // s1 right point sign
    if (lsign * rsign > 0) // s1 endpoints have same sign relative to
s2
        return FALSE;      // => on same side => no intersect is
possible
    // the segments s1 and s2 straddle each other
    return TRUE;           // => an intersect exists
}
//==================================================================



// simple_Polygon(): test if a Polygon P is simple or not
//     Input:  Pn = a polygon with n vertices V[]
//     Return: FALSE(0) = is NOT simple
//             TRUE(1)  = IS simple
int
simple_Polygon( Polygon Pn )
{
    EventQueue  Eq(Pn);
    SweepLine   SL(Pn);
    Event*      e;                 // the current event
    SLseg*      s;                 // the current SL segment

    // This loop processes all events in the sorted queue
    // Events are only left or right vertices since
    // No new events will be added (an intersect => Done)
    while (e = Eq.next()) {        // while there are events
        if (e->type == LEFT) {     // process a left vertex
            s = SL.add(e);         // add it to the sweep line
            if (SL.intersect( s, s->above))
                return FALSE;      // Pn is NOT simple
```

```
            if (SL.intersect( s, s->below))
                return FALSE;      // Pn is NOT simple
        }
        else {                     // processs a right vertex
            s = SL.find(e);
            if (SL.intersect( s->above, s->below))
                return FALSE;      // Pn is NOT simple
            SL.remove(s);          // remove it from the sweep line
        }
    }
    return TRUE;      // Pn is simple
}
//==================================================================
```

References

I.J. Balaban, "An Optimal Algorithm for Finding Segment Intersections", Proc. 11-th Ann. ACM Sympos. Comp. Geom., 211-219 (1995)

Ulrike Bartuschka, Kurt Mehlhorn & Stefan Naher, "A Robust and Efficient Implementation of a Sweep Line Algorithm for the Straight Line Segment Intersection Problem", Proc. Workshop on Algor. Engineering, Venice, Italy, 124-135 (1997)

Jon Bentley & Thomas Ottmann, "Algorithms for Reporting and Counting Geometric Intersections", IEEE Trans. Computers C-28, 643-647 (1979)

Mark de Berg et al, Computational Geometry : Algorithms and Applications, Chapter 2 "Line Segment Intersection" (2000)

Tim Chan, "A Simple Trapezoid Sweep Algorithm for Reporting Red/Blue Segment Intersections", Proc. 6-th Can. Conf. Comp. Geom., Saskatoon, Saskatchewan, Canada, 263-268 (1994)

Bernard Chazelle & Herbert Edelsbrunner, "An Optimal Algorithm for Intersecting Line Segments in the Plane", Proc. 29-th Ann. IEEE Sympos. Found. Comp. Sci., 590-600 (1988)

Bernard Chazelle & Herbert Edelsbrunner, "An Optimal Algorithm for Intersecting Line Segments in the Plane", J. ACM 39, 1-54 (1992)

K.L. Clarkson & P.W. Shor, "Applications of Random Sampling in Computational Geometry, II", Discrete Comp. Geom. 4, 387-421 (1989)

John Hobby, "Practical Segment Intersection with Finite Precision Output", Comp. Geom. : Theory & Applics. 13

(4), (1999) [Note: the original Bell Labs paper appeared in 1993]

H.G. Mairson & J. Stolfi, "Reporting and Counting Intersections between Two Sets of Line Segments", in: Theoretic Found. of Comp. Graphics and CAD, NATO ASI Series Vol. F40, 307-326 (1988)

E. Myers, "An O(E log E + I) Expected Time Algorithm for the Planar Segment Intersection Problem", SIAM J. Comput., 625-636 (1985)

Joseph O'Rourke, Computational Geometry in C (2nd Edition), Section 7.7 "Intersection of Segments" (1998)

Franco Preparata & Michael Shamos, Computational Geometry: An Introduction, Chapter 7 "Intersections" (1985)

Michael Shamos & Dan Hoey, "Geometric Intersection Problems", Proc. 17-th Ann. Conf. Found. Comp. Sci., 208-215 (1976)