On Planning Immobilizing Grasps for a Reconfigurable Gripper

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Abstract: We propose a new reconfigurable gripper that consists of two parallel plates whose distance can be adjusted by a computer-controlled actuator. The bottom plate is a bare plane, and the top plate carries a rectangular grid of actuated pins that can translate in discrete increments under computer control. We propose to use this gripper to immobilize objects through frictionless contacts with three of the pins and the bottom plate. We present an efficient grasp planning algorithm, describe the design of the gripper, which is currently under construction, and report preliminary simulation experiments.

1 Introduction

Classical parallel-jaw grippers are unable to adapt to a wide variety of workpiece geometries; although dextrous hands have been proposed by the academic robotics community [10, 31], they are too expensive and cumbersome for typical manufacturing applications. Thus different grippers are used for different parts (hundreds of different models are indeed listed by gripper manufacturers). This calls for the design of reconfigurable grippers which combine the flexibility of dextrous hands with the cost-effectiveness and simplicity of parallel-jaw grippers, and for the development of accompanying software to reconfigure these grippers according to part geometry.

We propose in this article a new reconfigurable gripper that consists of two parallel plates whose distance can be adjusted by a computer-controlled actuator (Figure 1). The bottom plate is a bare plane, and the top plate carries a rectangular grid of actuated pins that can translate in discrete increments under computer control. We propose to use this gripper to immobilize objects through frictionless contacts with three of the pins and the bottom plate. Our approach is based on the notion of second-order immobility introduced by Rimon and Burdick [28]. We present an efficient algorithm for grasp planning, describe the design of the gripper, which is currently under construction, and report preliminary simulation experiments.

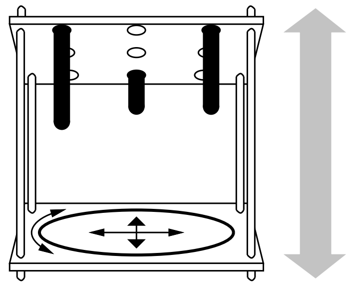

Figure 1. A reconfigurable gripper. An actuated pin is associated with each grid point. To avoid friction effects, the bottom plate should have three passive planar degrees of freedom.

1.1 Related Work

When a hand holds an object at rest, the forces and moments exerted by the fingers should balance each other so as not to disturb the position of this object. We say that such a grasp achieves *equilibrium*. For the hand to hold the object securely, it should also be capable of preventing any motion due to external forces and torques. Since screw theory [21] can be used to represented both displacements (*twists*) and forces and moments (*wrenches*), it is an appropriate tool for analyzing and synthesizing grasps. Indeed, it is known that six independent contact wrenches are necessary to prevent any infinitesimal displacement which maintains contact, and that a seventh one is required to ensure that contact cannot be broken [11, 32]. Such a grasp prevents any infinitesimal motion of the object, and it is said to achieve *form closure* [21, 27, 31]. A system of wrenches is said to achieve *force closure* when it can balance any external force and torque. Like wrenches and infinitesimal twists, force and form closure are dual notions and, as noted in [19, 20] for example, force closure implies form closure and vice versa.

The notions of form and force closure are the traditional theoretical basis for grasp planning algorithms. Mishra, Schwartz, and Sharir [18] have proposed linear-time algorithms for computing a finger configuration achieving force closure for frictionless polyhedral objects. Markenscoff and Papadimitriou [15] and Mirtich and Canny [16] have proposed algorithms for planning grasps which are optimal according to various criteria [8]. In each of these works, the grasp-planning algorithm outputs a single grasp for a given set of contact faces. Assuming Coulomb friction [20], Nguyen has proposed instead a geometric method for computing *maximal independent* two-finger grasps of polygons, i.e., segments of the polygonal boundary where the two fingers can be positioned independently while maintaining force closure, requiring as little positional accuracy from the robot as possible. This approach has been generalized to handle various numbers of fingers and different object geometries in [1, 5, 22, 24, 25, 26].

Robotic grasping and fixture planning are related problems (in both cases, the object grasped of fixtured must, after all, be held securely), but their functional requirements are not the same: as remarked by Chou, Chandru, and Barash [6], machining a part requires much better positional accuracy than simply picking it up, and the range of forces exerted on the parts are very different. The role of friction forces is also different: in the grasping context, where fingers are often covered with rubber or other soft materials, friction effects can be used to lower the number of fingers required to achieve form closure from seven to four; in the fixturing context, on the other hand, it is customary to assume frictionless contact, partly due to the large magnitude and inherent dynamic nature of the forces involved [6] (see, however [14] for an approach to fixture planning with friction). Finally, the kinematic constraints on the positions of the contacts are also quite different: in particular, dextrous grippers have *continuous* degrees of freedom, corresponding to the various finger joints, while modular fixtures have mostly *discrete* degrees of freedom, corresponding for example to the position of pins on an integer grid attached to a fixturing plate.

As noted by Wallack [35], there has recently been a renewed interest in the academic robotics community for manufacturing problems in general and fixturing in particular. Mishra has studied the problem of designing fixtures for rectilinear parts using toe clamps attached to a regular grid, and proven the existence of fixtures using six clamps [17] (this result has since then been tightened to four clamps by Zhuang, Goldberg, and Wong [36]). In keeping with the idea of *Reduced Intricacy Sensing and Control (RISC)* robotics of Canny and Goldberg [4], Wallack and Canny [34, 35] Brost and Goldberg [2] have recently proposed very simple modular fixturing devices and efficient algorithms for constructing form-closure fixtures of two-dimensional polygonal and curved objects. Brost and Peters [3] have extended this approach to prismatic three-dimensional objects, and Wagner, Zhuang, and Goldberg [33] have proposed a three-dimensional seven-contact fixturing device and an algorithm for planning form-closure fixtures of a polyhedron with pre-specified pose.

Recently, Rimon and Burdick have introduced the notion of *second-order immobility* [28, 29, 30] and shown that certain equilibrium grasps (or fixtures) of a part which do not achieve form closure effectively prevent any *finite* motion of this part through curvature effects in configuration space. They have given operational conditions for immobilization and proven the dynamic stability of immobilizing grasps under various deformation models [30]. An additional advantage of this theory is that second-order immobilization can be achieved with fewer fingers (four contacts for convex fingers) than form closure (seven contacts [11, 32]). In [23], we introduced a new approach to modular fixture planning, based on the notion of second-order immobility. Here we bridge the gap between fixture and grasp planning by using second-order immobility to design efficient grasping strategies for a new class of reconfigurable grippers with mostly discrete degrees of freedom, which have the potential of achieving the same level of flexibility as dextrous robotic hands for a fraction of the cost.

1.2 Second-Order Immobility

Let us consider a rigid object and the contacts between d pins and this object. Let us also denote by $\boldsymbol{p}_i$ $(i = 1,..,d)$ the positions of the contacts in a coordinate frame attached to the object, and by $\boldsymbol{n}_i$ $(i = 1,..,d)$ the unit inward normals to the corresponding faces. Equilibrium is achieved when the contact wrenches balance each other, i.e.,

$$\sum_{i=1}^{d} \lambda_i \begin{pmatrix} \boldsymbol{n}_i \\ \boldsymbol{p}_i \times \boldsymbol{n}_i \end{pmatrix} = 0, \tag{1}$$

for some $\lambda_i \geq 0$ $(i = 1,..,d)$ with $\sum_{i=1}^{d} \lambda_i = 1$. Equilibrium is a necessary, but not sufficient, condition for force and form closure.

Czyzowicz, Stojmenovic and Urrutia have recently shown that three contacts in the plane and four contacts in the three-dimensional case are sufficient to immobilize (i.e., prevent any *finite* motion of) a polyhedron [7]. Rimon and Burdick have formalized the notion of immobilizing grasps and fixtures in terms of isolated points of the free configuration space [28, 29, 30].

They have shown that equilibrium fixtures that do not achieve form closure may still immobilize an object through second-order (curvature) effects in configuration space: a sufficient condition for immobility is that the *relative curvature form* associated with an essential equilibrium[1] grasp or fixture be negative definite. The relative curvature form can be computed in terms of the contact positions as well as the surface normals and curvatures of the body and pins at the contacts.

In the case of equilibrium contacts between spherical pins and polyhedra, it is easily shown [23] that the symmetric matrix associated with the relative curvature form is

$$\mathcal{K} = \sum_{i=1}^{d} \lambda_i \{([\boldsymbol{n}_{i\times}]^T[\boldsymbol{p}_{i\times}])^S - r_i[\boldsymbol{n}_{i\times}]^T[\boldsymbol{n}_{i\times}]\}, \quad (2)$$

where r_i denotes the pin's radius, the weights λ_i are the equilibrium weights of (1), and, by definition, $\mathcal{A}^S = \frac{1}{2}(\mathcal{A} + \mathcal{A}^T)$.

Thus, immobilizing grasps can be found by enumerating all equilibrium configurations, then testing that the matrix $\mathcal{K}$ is negative definite.

2 Grasp Planning

In this section, we present an efficient algorithm for enumerating all immobilizing grasps of a polyhedral object. To simplify this planning process, we reduce the problem of achieving contact between a spherical pin and a plane to the problem of achieving point contact with a plane. This is done without loss of generality by growing the object to be fixtured by the pin radius and shrinking the spherical end of the pin into its center (see [2, 34, 35] for similar approaches in the two-dimensional case).

As shown in Section 2.2, when an object is immobilized by the gripper, its orientation relative to the gripper depends only on the position of the pins and not on their length. This allows us to decompose the grasp planning algorithm into three steps as follows.

For each quadruple of faces do:

1. Test whether they can be held in essential equilibrium.

2. Enumerate all pin positions that may immobilize the object and compute the corresponding object orientation.

3. For each such position, enumerate the pin lengths that immobilize the object and compute the remaining grasp parameters.

[1]Essential equilibrium is achieved when the coefficients λ_i in (1) are uniquely defined and strictly positive [28].

2.1 Testing Essential Equilibrium

For a polyhedral object, the normals $\boldsymbol{n}_i$ are fixed vectors. To ensure essential equilibrium, we restrict our attention to quadruples of faces such that no three of them have coplanar normals. This ensures that the coefficients λ_i in (1) are uniquely defined, and it allows us to compute them from the equation $\sum_{i=1}^{3} \lambda_i \boldsymbol{n}_i = 0$ and to test whether they all have the same sign. If they do not, the four candidate faces are rejected.

Our gripper can be used to immobilize a polyhedral object through contacts with three of the top plate pins, and either a face, an edge-and-vertex, or a three-vertex contact with the bottom plate. Let us assume for the sake of simplicity that the faces of the polyhedron are triangular (convex faces can be handled in similar ways, see [6] for a related approach). Any wrench exerted at a contact point between a face and the bottom plate can be written as a positive combination of wrenches at the vertices. Likewise, the wrenches corresponding to an edge-and-vertex contact are positive combinations of wrenches exerted at the end-points of the line segment and at the vertex. Thus equilibrium configurations can be found, in general, by writing the equilibrium equation (1) for six elementary wrenches.

We detail the case of a contact between the bottom plate and a triangular face with unit normal $\boldsymbol{n}$ and vertices $\boldsymbol{v}_i$ $(i = 1, 2, 3)$. Let $\boldsymbol{p}_i$ and $\boldsymbol{n}_i$ $(i = 1, 2, 3)$ denote the remaining contact points and surface normals; we take advantage of the fact that the overall scale of the wrenches is irrelevant to rewrite (1) as

$$\begin{cases} \sum_{i=1}^{3} \lambda_i \begin{pmatrix} \boldsymbol{n} \\ \boldsymbol{v}_i \times \boldsymbol{n} \end{pmatrix} + \sum_{i=1}^{3} \mu_i \begin{pmatrix} \boldsymbol{n}_i \\ \boldsymbol{p}_i \times \boldsymbol{n}_i \end{pmatrix} = 0, \\ \lambda_1 + \lambda_2 + \lambda_3 = 1, \end{cases} \quad (3)$$

where $\lambda_i, \mu_i \geq 0$ $(i = 1, 2, 3)$.

We can parameterize each contact $\boldsymbol{p}_i$ by two variables u_i, v_i. For example, we can use the parameterization

$$\boldsymbol{p}_i = (\, u_i, v_i, a_i u_i + b_i v_i + c_i \,)^T, \quad (4)$$

where the coordinates of $\boldsymbol{p}_i$ are written in some coordinate system attached to the object with z axis parallel to the vector $\boldsymbol{n}$, and a_i, b_i, c_i are constants. Assuming convex faces, the fact that the contact points actually belong to the faces can be written as a set of linear inequalities on u_i, v_i:

$$f_{ij}(u_i, v_i) \leq 0, \quad j = 1, .., k_i, \quad (5)$$

where k_i is the number of edges that bound face number i.

When the four surface normals are linearly independent, the equation $\boldsymbol{n} = -\sum_{i=1}^{3} \mu_i \boldsymbol{n}_i$ allows us to compute the coefficients μ_i and check whether they have the same sign. If they do not, the quadruple of faces under consideration is rejected. If they do, the (3) provides four linear equations in the the nine unknowns λ_i, u_i, v_i $(i = 1, 2, 3)$. We test the existence of equilibrium configurations by using linear programming to determine whether the five-dimensional polytope defined by (3) and the inequality constraints (5) and $\lambda_i \geq 0$ is empty. When this polytope is not empty, there is only (in general) a subset of each face that can participate in an equilibrium configuration. The subset corresponding to face number i is determined by projecting the polytope defined onto the plane (u_i, v_i). Several algorithms can be used to perform this projection, including Fourier's method [9], the convex hull and extreme point approaches of Lassez and Lassez [13, 12], and the Gaussian elimination and contour tracking techniques of Ponce *et al.* [26]. For faces with a bounded number of edges, all of these algorithms run in constant time, and they can be used to construct subsets of the original faces that are then passed as input to the rest of the algorithm. This projection process affords an early pruning of gripper configurations that cannot achieve equilibrium and therefore immobilization.

2.2 Enumerating Pin Positions

Let us now suppose for a moment that we have chosen the configurations (position plus length) of the three pins, and let $\boldsymbol{q}_i$ $(i = 1, 2, 3)$ denote the corresponding position of their tips in the coordinate system attached to the gripper. If $\mathcal{R}$ and $\boldsymbol{t}$ respectively denote the rotation of angle θ about $\boldsymbol{n}$ and the translation that map the gripper's coordinate frame onto the object's own frame,[2] we have

$$\boldsymbol{q}_i = \mathcal{R}\boldsymbol{p}_i + \boldsymbol{t}, \quad \text{for} \quad i = 1, 2, 3, \tag{6}$$

and substituting in (3) yields

$$\begin{cases} (\sum_{i=1}^{3} \lambda_i \boldsymbol{v}_i) \times \boldsymbol{n} + \sum_{i=1}^{3} \mu_i [\mathcal{R}^{-1}(\boldsymbol{q}_i - \boldsymbol{t})] \times \boldsymbol{n}_i = 0, \\ \lambda_1 + \lambda_2 + \lambda_3 = 1. \end{cases} \tag{7}$$

In turn, using the fact that $\boldsymbol{n} = -\sum_{i=1}^{3} \mu_i \boldsymbol{n}_i$, forming the dot product of the above expression with $\boldsymbol{n}$, and using elementary properties of triple products yields

$$\sum_{i=1}^{3} \mu_i [(\mathcal{R}^{-1}\boldsymbol{q}_i) \times \boldsymbol{n}] \cdot \boldsymbol{n}_i = 0. \tag{8}$$

[2]The translation $\mathbf{t}$ includes a "vertical" translation along $\mathbf{n}$ corresponding to the unknown distance between the two plates.

Note first that (8) is actually independent of the heights of the individual pins: indeed, since the rotation $\mathcal{R}$ is about the axis $\boldsymbol{n}$, the term $(\mathcal{R}^{-1}\boldsymbol{q}_i) \times \boldsymbol{n}$ only depends on the coordinates of $\boldsymbol{q}_i$ in the plane orthogonal to $\boldsymbol{n}$. For given pin positions, (8) is a univariate equation in θ, and its solution is easily shown to be $\theta = \operatorname{Arg}(C, S) + \pi$, where $\operatorname{Arg}(c, s)$ denotes the angle a such that $\cos a = c$, $\sin a = s$,

$$C = \sum_{i=1}^{3} \frac{\mu_i}{l_i}(a_i q_i + b_i r_i), \quad S = \sum_{i=1}^{3} \frac{\mu_i}{l_i}(-b_i q_i + a_i r_i),$$

and $l_i = \sqrt{1 + a_i^2 + b_i^2}$. (The proof is omitted here for the sake of conciseness.)

Thus we can first enumerate all possible pin locations on the lower plate and compute the corresponding rotations, then enumerate the corresponding pin heights and compute the corresponding object pose.

An exhaustive search of all possible grid coordinates would be extremely expensive: consider an object of diameter D (measured in units equal to the distance between successive grid points); there are a priori $O(D^4)$ different pin locations, since we can position one pin at the origin and the other two pins at arbitrary locations on the grid. Instead, we use an approach similar to the algorithms presented by Wallack and Canny [34, 35] and Brost and Goldberg [2], using bounds on the distance between two faces to restrict the set of grid coordinates under consideration. Clearly, each pin must lie within the horizontal projection of each face. Thus if we position the first pin at the origin, the integer point corresponding to the second pin is constrained to lie within the circular shell centered at the origin with inner radius equal to the minimum distance between the projections of the two corresponding faces and outer radius equal to the maximum distance. Given the position of the second pin, the third pin is now constrained to lie within the region formed by the intersection of the two shells associated with the first and second pin.

Enumerating the pin locations thus amounts to determining the integer positions falling in planar regions defined by a circular shell or the intersection of two such shells. This can be done in optimal time proportional to the number V of these points by using a scan-line conversion algorithm (Figure 2).

2.3 Enumerating Pin Lengths

Once the position of the pins has been chosen and the corresponding rotation has been computed, we can align the gripper's and object's coordinate systems so

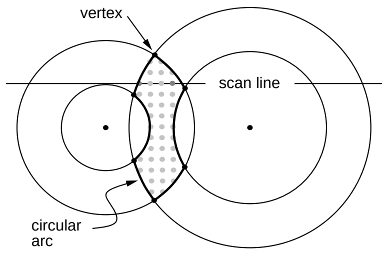

Figure 2. Scan-line conversion: spans between consecutive boundary elements are filled one scan-line at a time.

they are only separated by the translation $\boldsymbol{t}$. In particular, (4) and (6) reduce to

$$\begin{cases} u_i + x = q_i \\ v_i + y = r_i \\ a_i u_i + b_i v_i + c_i + z = s_i \end{cases} \quad \text{for} \quad i = 1, 2, 3, \quad (9)$$

where $\boldsymbol{t} = (x, y, z)^T$ and $\boldsymbol{q}_i = (q_i, r_i, s_i)^T$. In particular, let us denote by f_i the corresponding face and by f_i' the set $\{(x, y) = (q_i - u_i, r_i - v_i) | f_{ij}(u_i, v_i) \leq 0 \text{ for } j = 1, .., k_i\}$.[3] The first two rows of (9) imply that the horizontal part of the translation is restricted to lie within the polygon $F = f_1' \cap f_2' \cap f_3'$. Substituting in the third row of (9) and using the fact that $q_1 = r_1 = s_1 = 0$ now yields

$$\begin{pmatrix} s_2 \\ s_3 \end{pmatrix} \in \{\mathcal{A} \begin{pmatrix} x \\ y \end{pmatrix} + \boldsymbol{b} | \begin{pmatrix} x \\ y \end{pmatrix} \in F\}$$

where

$$\mathcal{A} = \begin{pmatrix} a_1 - a_2 & b_1 - b_2 \\ a_1 - a_3 & b_1 - b_3 \end{pmatrix}, \; \boldsymbol{b} = \begin{pmatrix} a_2 q_2 + b_2 r_2 + c_2 - c_1 \\ a_3 q_3 + b_3 r_3 + c_3 - c_1 \end{pmatrix}.$$

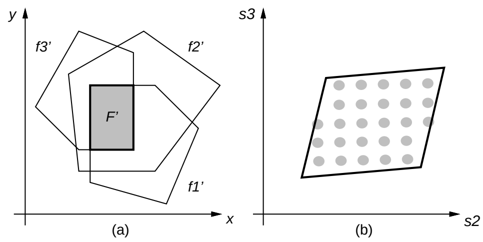

Figure 3. Enumerating pin lengths: (a) the polygon F' defined in the x, y plane by the intersection of the faces f_i', and (b) the corresponding convex polygon in the h_2, h_3 plane, along with the integer points inside it.

[3] The set f_i' is simply the convex polygon obtained by projecting f_i onto the (u_i, v_i) plane, then applying to the projection a symmetry with respect to the origin and a translation by (q_i, r_i).

In other words, the possible values of (s_2, s_3) are simply the integer points that lie in the polygon defined by the above equation, which is obtained from F by an affine transformation. These points can once again be determined in optimal time proportional to their actual number using a polygon scan-line conversion algorithm.

Now, for a given configuration (location plus length) of the pins, (9) forms a system of nine linear equations in the six variables u_i, v_i $(i = 1, 2, 3)$ and the three components of $\boldsymbol{t}$. This system is readily solved to yield the pose of the object and the separation of the plates. Note that the values of the coefficients λ_i are easily computed from (7) if required.

2.4 Algorithm Analysis

Let us assume without loss of generality that each face can be inscribed in a disc of diameter d (note that $d \leq D$ and that in practice, we will often have $d \ll D$). The area of a circular shell is then $O(Dd)$, and the area of the intersection of two such shells is also at worse $O(Dd)$. Finally, the area of the polygon F' is $O(d^2)$. Thus the total complexity of the algorithm is $O(N^4 D^2 d^4)$. To obtain a more realistic estimate of our algorithm's behavior, let us consider a polyhedron with total area A whose faces all have the same area, so $d^2 = O(D^2/N) = O(A/N)$. Under this assumption, the complexity of our algorithm is $O(N^2 A^3)$. It should also be noted that in practice, when $d \ll D$, the area of the intersection of two circular shells will often be proportional to d^2 rather than Dd. Of course, this does not change the worst-case complexity of the algorithm.

3 Gripper Design

The current design is a straightforward adaptation of the conceptual design shown in Figure 1. A front view is shown in Figure 4: the gripper consists of two sub-assemblies: the top plate assembly and the lower plate assembly. The top plate assembly consists of a grid of 36 linear actuators (only 6 are seen in the front view of the gripper in Figure 4) of 0.75 inch diameter each and spaced an inch apart. A gripper pin is attached to one end of the lead screw of each actuator. This enables individual control for each gripper pin. Each gripper pin is also mounted with a load cell that functions as a force sensor capable of measuring the axial force on the pin (although position control is used in the current application). The top and lower plate assemblies can be moved relative to each other using a large linear actuator (Figure 4).

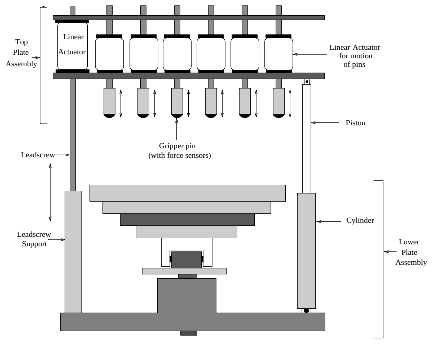

Figure 4. A front view of the reconfigurable gripper.

The lower plate assembly has three passive degrees of freedom and supports the object to be grasped. The detailed design of the lower plate is shown in Figure 5. The three degrees of freedom are provided to overcome friction between the object and the object support plate. The two translational degrees of freedom are achieved through two linear crossed roller slides. These slides are designed to ensure precise, uniform, linear motion with low friction and minimum side play even under high load (> 75 lbs) conditions. They are mounted orthogonal to each other (Figure 5) and are equipped with stops to ensure that the slides do not come apart. The additional rotational degree of freedom is achieved by mounting the two translational units on top of a shaft that is guided by a roller bearing as shown in Figure 5. The bottom end of the shaft is attached to a teflon ring that comes in contact with a stainless steel ball. This mechanism ensures that the shaft does not rub against the bottom surface. The height of the ball can be adjusted by a teflon stop from the bottom of the bushing.

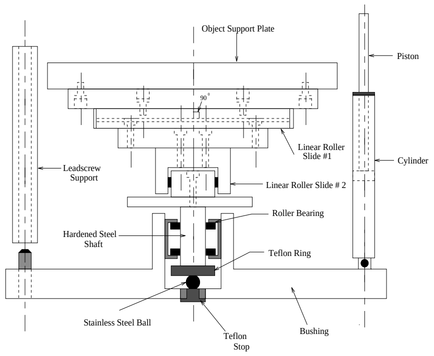

Figure 5. Detailed design of the lower plate assembly.

We have completed the mechanical assembly of the gripper (Figure 6), and are in the process of completing the electronics and computer interface. It should be noted that the prototype under construction is intended for proof-of-concept experiments only: we plan to design and construct in the near future a second-generation gripper with much fewer actuators, taking advantage of the fact that only three of the pins may have to move simultaneously.

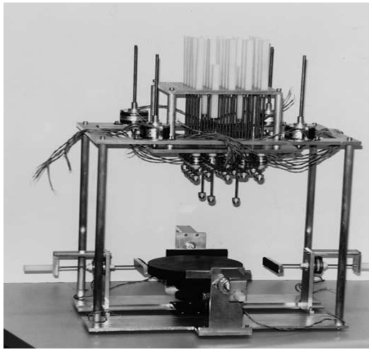

Figure 6. The prototype of the gripper.

4 Implementation and Results

Since our gripper is not operational yet, we can only present simulated grasping experiments. The implementation has been written in C. Figures 7 and 8 show some of the grasps of a tetrahedron and of a polyhedron with 10 faces that our algorithm has found using a 5×5 grid.

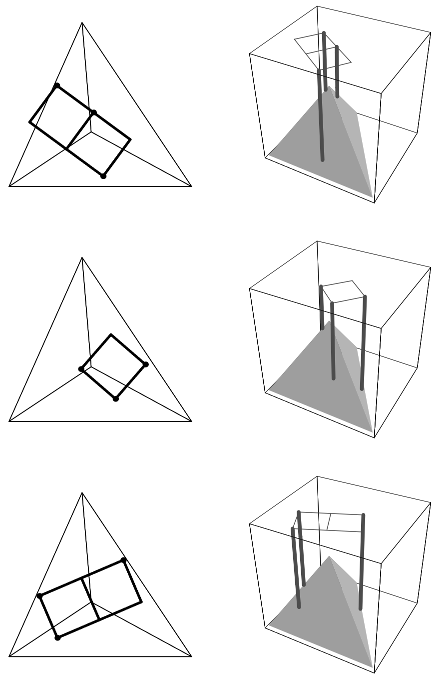

Figure 7. Grasping a tetrahedron.

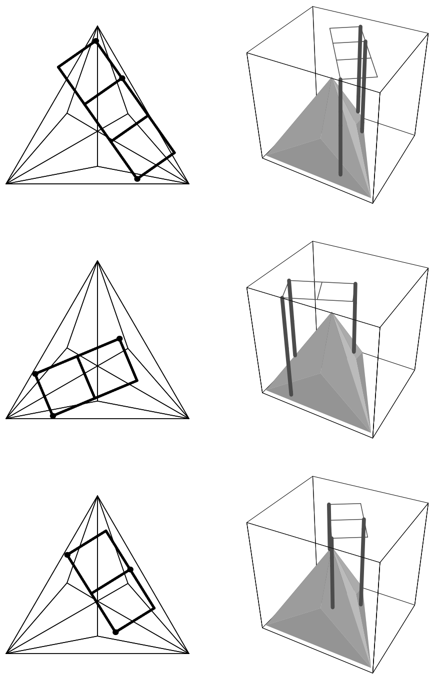

Figure 8. Grasping a 10-face polyhedron.

Table 1 gives some quantitative results. We have used a $K \times K$ grid with various values of K, as well as pins whose height may take ten discrete values. The table shows the results obtained without any pruning (N), using circular shell pruning only (S), and combining the projection- and shell-pruning stages (P+S). All run times have been measured on a SUN SPARCstation 10.

Tetrahedron							
K	Number of Solutions	Run Time (s)			# Candidates		
		N	S	P+S	N	S	P+S
3	0	1	1	1	33	10	10
4	160	1	1	1	141	42	40
5	704	2	1	2	411	145	135
6	1,963	4	2	2	927	391	378
7	4,263	8	4	4	1,839	795	751

Polyhedron with 10 Faces							
K	Number of Solutions	Run Time (s)			# Candidates		
		N	S	P+S	N	S	P+S
3	0	20	1	2	2,772	750	712
4	189	47	3	4	11,844	2,213	2,102
5	794	72	9	9	34,524	3,819	3,537
6	2,326	142	20	20	77,868	7,811	7,125
7	5,046	341	43	41	154,476	16,259	14,951

Table 1. Quantitative results for two test objects.

The table shows that, as could be expected, pruning eliminates a much larger percentage of the possible configurations in the case of the polyhedron with 10 faces than in the case of the tetrahedron, corresponding to the fact that, for most choices of faces, the range between the minimum and maximum distances is smaller for the polyhedron with 10 faces.

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