

The Use of Explicit Building Blocks in Evolutionary Computation

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This paper proposes a new algorithm to identify and compose building blocks. Building blocks are interpreted as common subsequences between good individuals. The proposed algorithm can extract building blocks from a population explicitly. Explicit building blocks are identified from shared alleles among multiple chromosomes. These building blocks are stored in an archive. They are recombined to generate offspring. The additively decomposable problems and hierarchical decomposable problems are used to validate the algorithm. The results are compared with the Bayesian Optimization Algorithm, the Hierarchical Bayesian Optimization Algorithm, and the Chi-square Matrix. This proposed algorithm is simple, effective, and fast. The experimental results confirm that building block identification is an important process that guides the recombination procedure to improve the solutions. In addition, the method efficiently solves hard problems.

Keywords: Genetic Algorithm; Estimation of Distribution Algorithm; Building Block; Linkage; Multi-parent Recombination

Introduction

Evolutionary computation (EC) is a class of algorithms that are suitable for problems with a very large search space (Yu and Gen 2010). The classic algorithm in EC is the genetic algorithm (GA) (Goldberg 1989), which was inspired by the process of genetic breeding in nature. Genetic algorithms have four main processes. First, the solution is encoded into a specific pattern, such as a binary string, called a chromosome or an individual and a number of encoded solutions, called a population, are generated. Second, good chromosomes are selected from the population. Third, selected chromosomes are mated using a crossover operator. Fourth, mated chromosomes are mutated. The fourth process is an optional step. New offspring are produced after these processes are completed.

As in nature, the evolution process will improve solutions from generation to generation. The EC algorithms are based on the premise that better solutions can evolve through selection and genetic operations (Goldberg and Sastry 2010).

In GAs, the solutions are improved based on the assumption that substructures of the good solutions exist in the good individuals. If the substructures can be identified and combined correctly, the new solutions will be better (Goldberg and Sastry 2010). The schema theorem (Schaefer 2007) tells us that the properties of good substructures are “short, low-order, highly-fit.” These substructures are called building blocks (BBs).

The building block hypothesis (BBH) states that the short, low-order, highly-fit schemata recombine to form higher order schemas of conceivably higher fitness. This hypothesis is used to explain the mechanism of GAs. To perform the recombination, a crossover operator is used. The crossover operation was claimed to construct rather than disrupt BBs (Schaefer 2007). Generally, one-point crossover is sufficient to solve problems. However, several crossover methods have been introduced to improve the effectiveness of the recombination, such as the two-point, multiple-point, and uniform crossover methods (Sivanandam and Deepa 2008). Different methods work well for different types of problems. The mutation operation was reported to provide a new source of genetic material (Schaefer 2007). However, setting the appropriate mutation rate is difficult. We only know that it should be low. Conversely, the crossover rate should be high (Lobo, Lima and Michalewicz 2007; Eiben and Smith 2011). The canonical Genetic Algorithm is called the simple GA. It uses fitness-proportion selection, single-point crossover and one-bit mutation. The simple GA algorithm is easy to use, but it cannot solve hard problems effectively (Sivanandam and Deepa 2008). However, several GAs have been successfully applied to a wide range of real world problems, such as Wangka (2009), Cheung, Cheung, Tobar, Caram and Garcia (2011), Alabsi and Naoum (2012), and Ozcan and Eshaf (2013).

Multiple parent recombination operators have been studied (Eiben, Rauee and Ruttkay 1994; Eiben 2000; Ting and Chen 2007). Rather than limiting the gene from two parents, the gene pool from the population was used. Some forms of explicit blocks of genes that come from multiple parents were introduced in many pioneering works (Syswerda 1993; Smith and Fogarty 1995; Levenick 1995). These “blocks” were used by recombination operators to compete for a position over certain loci. In Syswerda (1993), an explicit model was created by counting the number of alleles in each bit position contributed by the selected population. This information was used to perform recombination of genes. Smith and Fogarty (1995) used the linkage between blocks of genes of varying size to generate offspring.

A parallel concept to Building Blocks is linkage learning. Linkage is a relation between decision variables. While Building Blocks are defined as a group of genes, linkage emphasizes more on the relationship between genes. One important factor in the success of Evolutionary algorithms is a good coding scheme that puts genes in the same building blocks together thus providing tight linkage. Many linkage learning methods have been proposed to discover good building blocks, for example, linkage learning genetic algorithm (LLGA) and its variance (Chen and Goldberg 2002; Chen 2006; Chen and Lim 2008). A good survey of the linkage learning can be found in (Chen, Yu, Sastry and Goldberg 2007).

If the BBs are mixed properly, the quality of the result will be improved (Goldberg and Sastry 2010). Therefore, the BBs must be identified. Building Block Identification (Aporntewan and Chongstitvatana 2005) is a method that explicitly identify building blocks using correlation between genes. This knowledge is then used to partition genes into groups. Once the building blocks are identified, the appropriate crossover is used to recombine BBs. Although there are many ways to identify the

building blocks, a majority of the methods are Estimation of Distribution Algorithms (EDAs) (Larrañaga and Lozano 2001; Pelikan, Goldberg, and Lobo 2002; Pelikan, Sastry, and Cantú-Paz 2006). The algorithms that employ some data structure (so called “model”) to organize the decision variables are also considered to use linkage. They include the probabilistic model-building genetic algorithms (PMBGAs) or EDAs (Coffin and Smith 2008) where the probabilistic models are used to represent relationship of decision variables. In the EDAs, knowledge is shared using a distribution model for the population, and new offspring are sampled from the model. However, most EDAs require some knowledge to identify the relationships between individuals in a population and to build a model. Because BBs are explicitly extracted in terms of probabilistic models, EDAs can solve hard problems effectively (Larrañaga, and Lozano 2001; Pelikan, Sastry, and Cantú-Paz 2006). The knowledge sharing in both the model building and model sampling processes is the main advantage of the EDA method.

The messy GA (mGA) is another approach that uses explicit BBs in the early stage (Goldberg, Korb and Deb 1989; Goldberg, Deb, Kargupta and Harik 1993). The messy GA worked by building up increasingly longer and better strings from shorter building blocks. It progressed in two phases: primordial and juxtapositional. The first phase built up the population of small and highly fit candidate. The second phase put them together to form solutions. The juxtaposition is performed by “cut” and “splice” operators that work on messy encoding similar to the crossover operators work on a fixed length string. A newer generation of mGA is the gene expression messy Genetic Algorithm (gemGA) (Kargupta 1996) that simplified the encoding of strings and incorporate linkage learning phase which resulted in a large reduction of running time. A recent study proposed a simplified BB, called a “Fragment” (Sangkavichitra and Chongstitvatana 2010). The substructures of BBs are defined as the substructures that two highly fit individuals have in common. These substructures are the Fragments. According to the BBH, the Fragments are the BBs because they can be recombined into higher order BBs (Goldberg and Sastry 2010). The simple GA with Fragment Crossover (sGA-FC) method is introduced. It is based on the simple GA and a special crossover (Sangkavichitra and Chongstitvatana 2010). The Fragment Crossover (FC) tries to maximize the schema exchange using multiple-point crossover scheme.

In this paper, we propose a new simple method that was inspired by multi-parent recombination, which can share alleles among multiple chromosomes, and EDAs, which can identify and compose the BBs explicitly. The proposed method is named the “Building Blocks Identification and Composition” (BBIC) algorithm. Two well-known BB validation problems were used to demonstrate the behavior of the proposed algorithm. The capabilities of the BBIC algorithm were benchmarked using a set of hard problems (Collard, Gaspar, Clergue and Escazut 1998; Finger, Stutzle and Lourenco 2002). The results were compared with the sGA-FC, the Chi-square Matrix (CSM) (Aporntewan and Chongstitvatana 2004), the Bayesian Optimization Algorithm (BOA) (Pelikan 2005), and the Hierarchical Bayesian Optimization Algorithm (hBOA)

(Pelikan 2005). The proposed method outperforms these algorithms in terms of the number of function evaluations and the execution time.

The paper is organized as follows. The Building Block definition is presented in Section 2. Section 3 describes the BBIC algorithm. The experiment and results are explained in Section 4. The benchmark problems and results are presented in Section 5. Section 6 discusses a plausible mechanism that explains the behavior of the algorithm. Finally, concluding remarks about the proposed method are given in Section 7.

Building Block

Generally, GAs are simple to use but the underlying mechanism is difficult to understand. The schema theorem was proposed to explain the effect of each operator (selection, crossover, and mutation) on the evolution. The effect of recombination process on BBs is not easily understood because it has an indirect impact on BBs. This paper introduces a simpler form of the Building Block. The proposed Building Block is defined as a contiguous substructure of a chromosome, and is a part of a subschema. The defined Building Block can be interpreted as a basic form of the explicit subschema. Throughout this article, we will use this definition of Building Block. The BBs in a chromosome can take many possible patterns, as shown in Figure 1. An example of a BB is shown in Figure 2. The minimum size of a BB is one allele and, the maximum size is equal to the length of the chromosome.

[Figure 1 here]

[Figure 2 here]

BBIC: Building Blocks Identification and Composition algorithm

In this section, we describe the proposed algorithm. The BBIC algorithm consists of two main parts: Building Block identification (BBI) and Building Block composition (BBC). The BBI process extracts the substructures that are common between good individuals (selected chromosomes), and the BBC process assembles them to create new offspring. The main objective of the BBIC algorithm is to capture and combine the sharing knowledge between good individuals. The main advantages of the algorithm are its simplicity and its performance.

Building Block Identification

Typically, GAs do not explicitly recognize BBs. The main operator that acts on BBs is the crossover, which processes BBs in an implicit manner (Schaefer 2007). There are many crossover methods, each suitable for a different type of problem. Most of the methods recombine the schemata without knowledge about the BBs. However, they work well because the crossed chromosomes are good solutions that have passed a

selection process and are expected to contain the ideal BBs (Goldberg and Sastry 2010). The crossover operation mixes the chromosomes. Normally, these methods differ in the number of cross points and the number of parents. In the exchange, the crossover operator only manipulates the substructures or alleles that differ between chromosomes. The similar substructures remain in the same place, only the distinct parts are moved or separated. This exchange mechanism causes schema disruption. The different patterns of disruption in each crossover method bias the results in different ways (Smit and Eiben 2010). Substructures that are common between chromosomes are unchanged during crossover. The common substructures are candidate BBs. We consider the similarity of bits that are in the same position of two chromosomes. This information can reveal the boundary of the potential BBs. The distinct parts should not be disturbed because they might be part of the BBs. We define the BB as the substructures that are common and uncommon between any two chromosomes. Both types of BB are retained to maintain the diversity in each generation. Evolution mechanisms (selection and recombination) process these BBs to improve the quantity and quality of the solutions.

[Figure 3 here]

The BBI process is illustrated by an example in Figure 3. Note that the string is indexed from left to right starting at position 1. Given two 10-bit chromosome sequences, $C1 = (1,0,1,1,0,1,0,1,0,1)$ and $C2 = (1,1,1,1,0,0,1,0,0,1)$, the BBs of $C1$ and $C2$ are $B1 = (1)$, $B2,1 = (0)$, $B2,2 = (1)$, $B3 = (1,1,0)$, $B4,1 = (1,0,1)$, $B4,2 = (0,1,0)$, and $B5 = (0,1)$ consecutively.

A common schema is defined as any identical contiguous parts between two chromosomes, and the uncommon schema is defined as the difference. The BB can be interpreted as contiguous subsequences that are common and uncommon between two chromosomes. The length of each BB must be greater than or equal to one bit. The definitions of the order and length of the BB are similar to those used in GAs. The order of a schema is the number of fixed bit positions. The BB comprises all of the fixed bit positions; therefore, the size of a BB is the order, e.g., in Figure 3, $B3 = (1,1,0)$ and $order(B3) = size(B3) = 3$. The length of a schema is the distance between the first and last fixed bit positions, e.g., in Figure 3, $B3$ begins at position 3 and ends at position 5 and $length(B3) = 5 - 3 = 2$. For BBs, the short and low-order schemata have a high potential to survive, and they will be recombined to create better solutions. The BBs can be considered as independent contiguous subschema that are tightly linked.

Building Block Composition

The knowledge sharing in the recombination process and the creation of new offspring process provides the main advantage of the multi-parent crossover GAs (Eiben 2002) and EDAs. These methods gain exploration power to find possible patterns of good solutions and enhance the quality of the solutions (Toussaint 2003). The BBIC

algorithm maintains the diversity of the BBs obtained in the identification process by keeping them in an archive. The building block composition demonstrates that the BBs are mixed in an explicit manner. There are many ways to compose BBs, but a simple method is sufficient. The BBs are selected one by one from the archive to create a new offspring. They are combined sequentially starting from the first position, as shown in Figure 7. In the archive, many BBs begin at the same position. If there is no guidance as to which BB is better, a random selection will avoid biases. When the first BB is selected for the first position, the next BB is concatenated to it, and the process is repeated until a new chromosome is formed. If there is no BB that starts at the current position, a random binary value (0 or 1) is used at that position. This situation might occur if the diversity of the population is too low because the population size is too small or the population is nearly converged.

An overview of the BBIC algorithm (Figure 4) is as follows. First, the good solutions are selected from the population. Based on the schema theorem and the BBH, the chance of survival for a schema increases in a chromosome that has fitness above average. Thus, the above average chromosome will be selected using the n -Best selection method (select n best individuals). Second (Figure 5), BBs are identified using the common schema partitioning criterion mentioned previously. In this step, every individual will be collated to explore every possible building block. Third, all BBs are labelled using their beginning position in the original individual. An example is shown in Figure 7. Building Block B2 begins at position 4 in the chromosome and is stored in an archive. Fourth (Figure 6), the BBs in the archive are randomly selected one by one to compose the new chromosome from the first position to the last position. This process is repeated until the new population is fulfilled.

[Figure 4 here]

[Figure 5 here]

[Figure 6 here]

The time complexity of the Building Block identification and the Building Block composition processes are $O((n)^2 \cdot l)$ and $O(n)$, respectively, where l is the chromosome length, and n is the number of selected chromosomes. There are two basic parameters, the population size and the number of selected chromosomes in the n -Best selection method. Compared to other GAs, or EDAs, it is easy to tune the algorithm. The mutation process is optional.

Figure 7 shows an example of the Building Block identification and the Building Block composition process. Two chromosome, $C1 = (1010101010)$ and $C2 = (1011011001)$, are compared using the common schema partitioning criterion. Six BBs are found. There are two common parts and two different parts between $C1$ and $C2$. The Building Blocks $B1 = (101)$ and $B4 = (10)$ are the common parts, and $B2 = (010)$, $B3 =$

(101), B5 = (10) and B6 = (01) are the different parts. The starting positions of Building Blocks B1 - B6 are 1, 4, 4, 7, 9 and 9. The BBs are labelled and archived into a composition table according to the beginning positions. The results are obtained once all pairs of the selected individuals have been collated. They are shown in the next composition table in Figure 7. The next process is Building Block composition, which creates new offspring for the next generation. In the example, two new chromosomes were produced, N1 and N2. Chromosome N1 was started by randomly selecting a Building Block for the position one. Building Block B31 was selected. The length of B31 is two, thus the next Building Block must begin at the position three. Building Block B12, B32 and B5 were randomly selected to complete N1. Chromosome N2 was created in the same manner as N1, but the last position has no Building Block in the archive. Therefore, a 0 or 1 bit is randomly generated.

[Figure 7 here]

Experimental Settings and Results

There are two parts of the experiment. The first part exhibits the behavior of BBs processing by comparing the BBIC algorithm to the sGA. The second part compares the performance of the BBIC algorithm to a group of competent algorithms. They are sGA-FC, CSM, BOA, and hBOA. The simple Genetic Algorithm (sGA) is used as the basic reference. The details of the first part of the experiments are described in the following sections.

Test Problems

Most of the problems used in the experiment are synthetic functions. The problems are classified into two categories: non-deceptive and deceptive. A deceptive problem lures the algorithm away from the ideal solution. Generally, deceptive problems are more difficult to solve than non-deceptive problems. It is difficult to claim what algorithm is suitable for a particular class of problems. Nevertheless, an experiment can be conducted to support the statement. In this experiment, the Royal Road function (non-deceptive) (Howard and Sheppard 2004) and the Trap-5 function (deceptive) (Beaudoin, Verel, Collard and Esczut 2006) were used. These two functions belong to the class of Additively Decomposable Functions (ADFs).

The Royal Road function was designed to test the ability of GAs to compose BBs. The general k -bit Royal Road is defined as

$$E_k(b_1, \dots, b_k) = \begin{cases} f & ; \text{if } u = k \\ 0 & ; \text{otherwise} \end{cases}, \quad (9)$$

where b_i is in $\{0,1\}$, $u = \sum_{i=1}^k b_i$ and $f = k$.

Additively decomposable functions, denoted by $E_{m \times k}$ are defined as

$$E_{m \times k}(k_1 \dots k_m) = \sum_{i=1}^m E_k(k_i), \quad k_i \in \{0,1\}^k. \quad (10)$$

Variables m and k are varied to produce a number of test functions. This problem is difficult because a hint about the BBs is not provided. The optimal solution is composed of all ones. This problem is representative of problems that have a simple BB structure.

The well-known Trap functions were designed to study BBs and linkage problems in GAs. The general k -bit trap functions are defined as:

$$E_k(b_1, \dots, b_k) = \begin{cases} f_{high} & ; \text{ if } u = k \\ f_{low} - ((u \times f_{low}) / (k - 1)) & ; \text{ otherwise} \end{cases} \quad (11)$$

where b_i is in $\{0,1\}$, $u = \sum_{i=1}^k b_i$ and $f_{high} > f_{low}$. Usually, f_{high} is set to k and f_{low} is set to $k-1$. The Trap problem is defined using equation (10).

The Trap functions fool the gradient-based optimizers to favor zeros, but the optimal solution is composed of all ones.

Measurement

To be able to visualize the behavior of BBs during processing, the BBs were classified into three classes: Pure BB, Mixed BB and Non BB. An example is shown in Figure 8. Pure BB means that the pattern in a chromosome corresponds to the ideal BBs in the problem. Mixed BB means that there is at least one allele (one bit) of the ideal BBs in the structure, thus Mixed BB can be regarded as a substantial source of genetic material in the recombination process (and also a source of diversity). Non BB means that no allele of the ideal BBs is in the structure. This class can be considered as a barrier to achieving the desired BBs. The number of BBs classified as Pure BB depends on the particular problem and its encoding length. Pure BB is used to indicate the performance of algorithms.

In the experiment, the binary encoding length of Royal Road and Trap-5 functions were 64 bits and 60 bits, respectively. The numbers of BBs classified as Pure BB were 8 and 12 for the Royal Road and Trap-5 problems, respectively. The number of BBs was measured in each generation over the entire population. For example, if a population size was 100 for the Trap-5 60-bit problem, there were 12 (BBs) x 100 (Chromosomes), equal to 1,200 BBs (Pure + Mixed + Non BBs), in each generation.

[Figure 8 here]

Building Blocks were used to illustrate the schema processing and the schema construction. There are two types of Building Blocks: common and difference (uncommon). The common BBs are the similar and contiguous bits found in two good chromosomes. The different BBs are uncommon contiguous bits found in good chromosomes. The numbers of common and different BBs were calculated from each Building Block identification operation. For example, there were two common BBs and four uncommon BBs in Figure 7. A change in numbers of the BBs in each type is an indicator of the behavior of the BBs during processing. For example, when the size of common BBs grows, it indicates that the BBs have been combined into larger BBs. This usually happens when the evolution has been converged to a local optimum. The number of BBs can be a good indicator of the diversity in the population. The ratio between the common and different BBs indicates the competition between alternate schemata.

Conditions

All tested problems were performed with 30 independent runs in both success and failure cases. All algorithms were required to find the optimal solution in all of the 30 runs for both cases. In the success cases, the minimum population size was set to achieve the optimum in all of the runs. The failure case, the maximum population size was reported. The numbers of Function Evaluation did not exceed one million for the Royal Road problem, and fifty thousand for the Trap-5 problem. The BBIC algorithm uses the n -Best selection and draws only the good chromosomes that have above average fitness. The maximum number of the selected chromosomes, or subpopulation, was limited to half of the population size. The sGA employs the tournament selection method, and the tournament-size was four. The crossover rate of the sGA was set to 1.0 for the best result. The mutation operator was not allowed in either algorithm because the aim of the experiment was to test the capability of the schema processing (the BB recombination), and thus it is better to avoid another source of genetic material. These parameter settings were the same for all test problems. In the failure case, the behavior of the sGA and the BBIC algorithm were shown only for the first one hundred generations.

The performance of GAs is compared using the number of Function Evaluation (#FEs). All results are averaged over the 30 runs using the same parameters setting. The parameters setting and results of the Royal Road and the Trap-5 problems are shown in Table 1.

[Table 1 here]

Note: #pop denotes the population size, #sub denotes the subpopulation (the selected individuals for identification process) and #FEs denotes the number of function evaluation.

Results

From the viewpoint of performance index (#FEs), the BBIC algorithm is inferior to the sGA in the Royal Road problem, but it outperforms the sGA in the Trap-5 problem. The population sizes in the success and failure cases are considered upper and lower bounds of the initial source of the diversity because there was no mutation or other source of genetic material during the process.

By design, the success case requires a minimum population size to achieve the optimal solution in all of the runs. This is regarded as an upper bound of the diversity that is required to obtain reliable results from the experiment. The failure case requires a maximum population size so that the optimal solution is not attained in all of the runs. This setting implies that if there is a larger population, the optimal solution can be found at least once. The population size in the failure case can be interpreted as a lower bound of the diversity that existed in the population. The behaviors of BBs during processing in both problems are discussed in the following paragraphs.

Behavior of BBs

The success of an algorithm is affected by the diversity of the population (to provide the genetic material). In the success case, the population size of the BBIC algorithm is larger than the sGA in the Royal Road problem, and it is smaller than the sGA in the Trap-5 problem. In the failure case, the population size of the BBIC algorithm is equal to the sGA in the Royal Road problem, and it is smaller than the sGA in the Trap-5 problem. The size of the subpopulation was set to the maximum value (half population size) in the failure case to provide the highest diversity. This setting guarantees the lower bound of the population size that cannot find the optimal solution in all of the runs. These facts indicate that the BBIC algorithm is better than the sGA at maintaining the diversity in the Trap-5 problem, and it is worse than the sGA in the Royal Road function.

The behavior of the sGA in the Royal Road problem is shown in Figure 9. The different types of BBs illustrate the change or transformation during the evolution process. Because the Royal Road problem is not deceptive, only the Pure BBs and the Mixed BBs compete in Figure 9(a,b). The number of Pure BBs shows the quality of the solution. There are eight BBs in this problem. In the success case, the number of Pure BBs increases continuously until the optimal solution is obtained because there is sufficient diversity. Figure 9(c,d) shows the convergence pattern of the entire population. In the failure case, the number of pure BBs is saturated at 5 from the 21th generation onward.

[Figure 9 here]

For the BBIC algorithm, the behavior is similar to the sGA (Figure 10(a-d)). There is a slight different in the failure case. This result can be observed in Figure 10b, which shows that the number of Mixed BBs is always higher than the number of pure BBs. The algorithm cannot learn enough to achieve good solutions, so the quality of the solutions in Figure 10d is inferior to the sGA (the BBIC algorithm achieved only 3 Pure BBs versus 5 pure BBs in the sGA).

[Figure 10 here]

The results from the Trap-5 problem are shown in Figures 11 and 12. In Figure 11, the overall behavior of the sGA for this problem is similar to Figure 9 except that there are Non BBs in the competition. This result is due to the deceptive impact of the Trap function.

[Figure 11 here]

For this problem, the BBIC algorithm performed better than the sGA because of the recombination power of the Building Block composition process. Comparing the quality of solutions of the sGA (Figure 11c) and the BBIC algorithm (Figure 12c), the number of pure BBs in the BBIC algorithm is higher than the sGA (for example, at the 10th generation). This is also noticeable in the failure case (BBIC in Figure 12b); the mixed BBs are maintained at a higher level than in the sGA (Figure 11b).

[Figure 12 here]

Competition among BBs

The data from the experiments on the BBIC algorithm are used to illustrate the competition among BBs. The average size of the BBs and the number of BBs are good indicators of the competition between common and different BBs. Figure 13(a-d) illustrates the competition behavior in the Royal Road problem. Figure 14(a-d) illustrates the Trap-5 problem. In both figures, CB denotes the average common Building Block size, DB denotes the average different Building Block size, #CB denotes the number of common BBs and #DB denotes the number of different BBs.

[Figure 13 here]

The average sizes of BBs in Figure 13(a-b) and the number of BBs in Figure 13(c-d) indicate the convergence. These two values are opposite. If the size of BBs increases, the number of BBs decreases. In the success case (Figure 13a), only the common BBs are gradually developed while the different BBs remain constant. In the

failure case (Figure 13b), the average size of the different BBs grows slightly because there are several mixed BBs (Figure 9b). The number of BBs reduces rapidly until the 31th generation at which point rate of reduction slows down (Figure 13d). The rates of convergence of the success and failure cases are different.

[Figure 14 here]

For the deceptive problem, there was a prominent competition between the structures of good and deceptive solutions. In the success case (Figure 14a), the average size of different BBs develops progressively because of the deceptive bias.

For both the Royal Road and the Trap-5 problems, in the success case, the required size of the subpopulation in the n -Best selection was approximately 10% of the population size. This small number results in fast convergence from the restricted diversity. Furthermore, the desired solution can be achieved from the selected chromosomes. This evidence supports the belief that among the good chromosomes there are good substructures. If the good structures can be identified correctly, they can be used to produce good results. Finding a large common Building Block is more difficult than finding a small one. Therefore, the BBs in the Royal Road problem (8-bit BB) are harder to identify than the BBs in the Trap-5 function (5-bit BB), and thus the Royal Road problem requires a larger population size than the Trap-5 problem. The collation of all pairs in the Building Block identification process provides a high degree of variation, and the recombination operation in the Building Block composition process generates a great number of possible patterns for the chromosome structure.

The number of BBs in each position in the archive varies (see Figure 7). Normally, the first position has more members because the Building Block identification process proceeds from left to right. When the Building Block size is bigger than a few bits, the next positions have fewer members because the next BBs have to be identified in sequence. This occurs repeatedly throughout the positions of the chromosome because the pattern of identification is from left to right.

Benchmark and Performance

In this section, several problems were used to test the performance of the BBIC algorithm. Most of them are BB validation problems. The results obtained using the BBIC algorithm are compared to the results obtained using several competing algorithms: the simple Genetic Algorithm (sGA), the sGA with Fragment crossover (sGA-FC), the Chi-square matrix (CSM), the Bayesian optimization algorithm (BOA) and the hierarchical Bayesian optimization algorithm (hBOA). The benchmark problems are the OneMax, the Royal Road, the Trap-5, the Hierarchical-If-and-only-If (HIFF) (Yu and Goldberg 2006), and the hierarchical Trap-1 (hTrap-1) function (Yu, Golberg, Sasty, Lima and Pelikan 2009). The details of these problems can be found in the references, and their characteristics are classified (Figure 15). Only the OneMax

problem is not a BB validation problem and is used as a performance reference for general cases. Both Trap-5 and hTrap-1 are deceptive problems, but Trap-5 has a straightforward BB structure, whereas hTrap-1 has a hierarchical BB structure. The HIFF problem is the only one that has a hierarchical BB structure and a multimodal fitness landscape, which has two optimal solution structures. The HIFF and the hTrap-1 problems are Hierarchical Decomposable Function (HDF) problems, which are harder to solve than Additively Decomposable Function (ADF) problems.

[Figure 15 here]

All of the benchmark problems were performed with 30 independent runs, and they were required to find the optimal solution in all of the runs. There are two versions of the BBIC algorithm: without mutation (BBIC-1) and with mutation, rate=0.02, (BBIC-2). We want to study how mutations affect the BBs and how they act as a source of genetically diverse material in each problem. The minimum population size used to achieve the optimum in 30 runs is shown in Table 2. The average number of function evaluations is compared to the sGA, sGA-FC, CSM, BOA and hBOA. The results are shown in Table 3. The results show that the BBIC-2 algorithm outperforms the BOA, hBOA and CSM in all of the problems, and the BBIC-1 algorithm outperforms all competitors in the Royal Road (except the 64-bit problem), the Trap-5, the HIFF and the hTrap-1 problems. The performance of the BBIC-1 algorithm (without mutation) on the tested problems, ordered from high to low, is as follows: OneMax, Royal Road, HIFF, Trap-5 and hTrap-1. In the same way, the performance of the BBIC-2 algorithm (with mutation) is ranked in the following order: OneMax, Royal Road, HIFF, hTrap-1, and Trap-5.

There are two parameter settings in the BBIC algorithm, as shown in Table 2. The population size indicates the level of diversity required to explore various solutions until the optimal solution is attained. The subpopulation denotes the level of selection pressure required to assure the quality of the result. The selection pressure is calculated as the size of the subpopulation (#sub) divided by the size of the population (#pop). A larger problem size or a harder problem requires that higher solution quality be obtained using a higher selection pressure. The diversity extension from the mutation process can compensate for the restricted variation of the population. The BBIC-2 algorithm requires a smaller population size and a smaller subpopulation size because of the mutation.

The results in Table 3 convince us that the proposed algorithm is suitable for ADF and HDF problems that are composed of tightly grouped BBs.

[Table 2 here]

(Note: #pop means population and #sub means subpopulation or selected chromosomes from the *n*-Best selection method. The bold face indicates the minimum value.)

[Table 3 here]

(Note: N/A denotes that the data were not available because the optimal solution cannot be found under the limited number of function evaluations ($\#FEs \leq 1,000,000$). The bold face indicates the best value.)

Discussion

The main component of the BBIC algorithm is the Building Block, which will be summarised in this section. There are two types of BBs: common and different. Many chromosomes in the selection process will be paired. The common BBs are regarded to have the same bias between two collated chromosomes. The identification procedure prevents the plausible unknown BBs from being disrupted. If the rate of construction of the BBs is higher than the rate of disruption of the BBs, the quality of the solutions will be improved. This is the main mechanism of the evolutionary process. The results of sGA-FCs and EDAs in several published papers confirm that various types of Building Block identification processes are useful (Larrañaga and Lozano 2001; Pelikan, Goldberg, and Lobo 2002; Pelikan, Sastry, and Cantú-Paz 2006). In regard to the use of explicit building blocks in GAs, the messy GA and its variants (Goldberg, Korb and Deb 1989; Goldberg, Deb, Kargupta and Harik 1993) are directly related to our work. The method of messy GAs is to improve the performance by increasingly build longer, highly fit strings from shorter building blocks. This is similar to how we compose BBs. However, our method is different in the identification of BBs and how BBs are stored and used.

The size of the BBs gives indirect information about the level of knowledge or diversity. If the size of the common BBs is longer, the size of the different BBs is shorter. This means that the evolution process has learned something about the models. The different BBs preserve the unexploited structures or diversity of the search space. In the early generations, the average size of the BBs is small because the diversity is high. Although the common BBs in the early stage are not reliable, the number of different BBs is twice the number of common BBs. The different BBs act as the choices for search space exploration. In the middle generations, the common BBs are more stable and more reliable. The different BBs act as the more distinct and more limited alternatives. In the later generations, the different BBs work as rare mutations because they are too short, and thus they have a very low chance to be selected.

Conclusion

The contiguous substructures of a chromosome can be regarded as the Building Blocks. They are identified from the mutual data between two chromosomes. The proposed BBs are a simple form of the explicit BBs because they are short, low-order and come from the highly-fit chromosomes. The BBIC algorithm uses the centralized knowledge, similar to EDAs, that all of the BBs are retained in an archive to create new offspring.

The Building Block composition process is simple and direct: it proceeds from left to right (first-bit to last-bit) to form a new chromosome using random selection from the archive.

The experimental results of the BBIC algorithm confirm that the identification of BBs is an important process that guides the recombination procedure to improve the solutions. The execution time of the Building Block identification and Building Block composition processes are $O((n)^2 \cdot l)$ and $O(n)$, respectively, where l is the chromosome length and n is the number of selected chromosomes. This is significantly less than the execution time of the CSM, BOA and hBOA. The proposed method is simple to implement and easy to tune. In addition, this method efficiently solves difficult problems of both the ADF and HDF classes.

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Table 1. Experimental parameter settings and results.

Parameters		Success 30 run				Failure 30 run	
		sGA		BBIC		sGA	BBIC
Problems	Size (bit)	#pop	#FEs	#pop : #sub	#FEs	#pop	#pop : #sub
Royal Road	64	1,200	11,900	2,000 : 200	13,437	200	200 : 100
Trap-5	60	2,300	28,400	1,000 : 100	7,834	300	100 : 50

Table 2. Benchmark parameter settings.

Problems	Problem size (bit)	BBIC-1 without mutation			BBIC-2 with mutation		
		#pop	#sub	(#sub / #pop)	#pop	#sub	(#sub / #pop)
OneMax	100	600	100	0.1666	100	10	0.1000
	150	1,000	150	0.1500	100	10	0.1000
	200	2,000	200	0.1000	100	10	0.1000
	250	3,000	200	0.0666	100	10	0.1000
Royal Road	64	2,000	200	0.1000	1,500	200	0.1333
	128	3,500	200	0.0571	2,500	200	0.0800
	256	5,500	300	0.0545	4,000	300	0.0750
Trap-5	100	2,000	200	0.1000	1,100	200	0.1818
	150	2,500	200	0.0800	2,000	200	0.1000
	200	4,500	200	0.0444	3,500	200	0.0571
	250	8,000	250	0.0312	6,700	200	0.0299
HIFF	32	200	100	0.5000	200	50	0.2500
	64	500	100	0.3000	500	100	0.2000
	128	2,000	200	0.1000	1,000	200	0.2000
	256	6,500	400	0.0615	6,000	300	0.0500
hTrap-1	27	300	100	0.3333	200	100	0.5000
	81	1,000	100	0.2000	1,000	100	0.1000
	243	11,500	300	0.0260	10,000	300	0.0300

Table 3. Benchmark results.

Problems	Problem size (bit)	Number of function evaluations (#FEs).						
		BBIC-1	BBIC-2	sGA	sGA-FC	CSM	BOA	hBOA
OneMax	100	7,500	1,700	22,200	3,800	14,000	5,100	N/A
	150	15,000	2,600	64,300	6,600	32,500	8,300	N/A
	200	32,000	3,400	128,300	9,800	60,000	12,500	N/A
	250	48,500	4,600	271,200	11,500	80,000	15,700	N/A
Royal Road	64	13,400	15,100	11,900	13,500	N/A	N/A	N/A
	128	28,700	31,800	49,500	29,900	N/A	N/A	N/A
	256	73,300	89,300	304,400	75,200	N/A	N/A	N/A
Trap-5	100	21,200	35,000	83,250	47,900	65,000	99,000	N/A
	150	31,300	73,000	305,500	114,000	165,000	220,000	N/A
	200	56,400	124,000	784,900	215,600	310,000	320,000	N/A
	250	102,900	219,000	N/A	375,900	750,000	490,000	N/A
HIFF	32	1,100	1,200	4,800	2,600	3,300	N/A	2,100
	64	4,700	4,900	38,800	11,800	14,500	N/A	7,800
	128	17,500	21,000	584,000	45,900	51,000	N/A	27,000
	256	71,300	118,000	N/A	222,700	370,000	N/A	90,000
hTrap-1	27	2,100	2,800	11,400	8,000	3,000	N/A	3,400
	81	8,900	9,500	N/A	N/A	35,000	N/A	30,000
	243	115,000	178,000	N/A	N/A	310,000	N/A	225,000

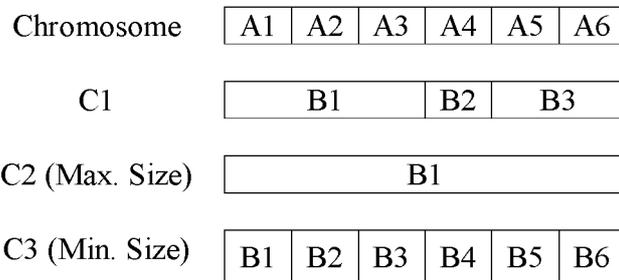


Figure 1. An example of possible patterns of BBs in a chromosome (A = Allele, C = Chromosome, B = Building Block).

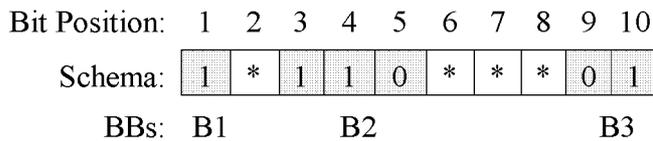


Figure 2. An example of BBs in a schema (B = Building Block).

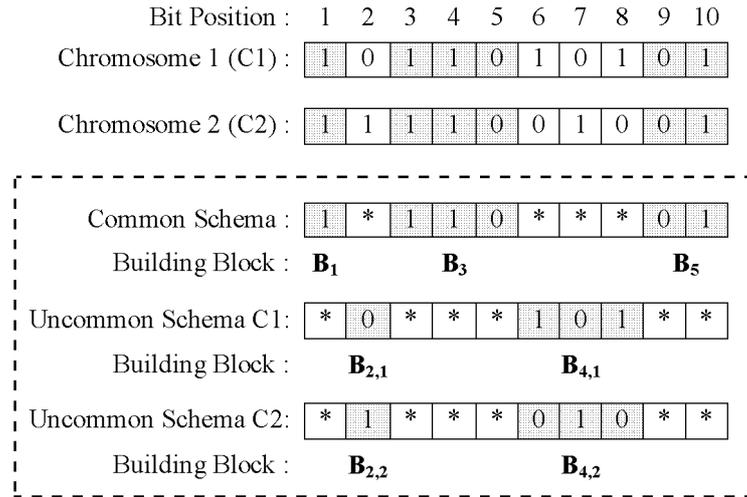


Figure 3. Building Block identification method.

D denotes a population.
 R denotes an individual (chromosome).
 B denotes common and uncommon subsequence (Building Blocks).
 T denotes an archive of common subsequences.

Algorithm BBIC {Building Block Identification and Composition algorithm}

$D_0 \leftarrow$ Generate R individuals (the initial population) at random

Repeat for $g = 1, 2, \dots$, until the stopping criterion is met

$D_{g-1}^N \leftarrow$ Select $N < R$ from D_{g-1} according to selection method.

$B \leftarrow$ Find the common and uncommon substructures (BBs) between D_{g-1}^N .
 {Building Block Identification}

$T \leftarrow$ Label B and record. {Building Block Composition}

$D_g^N \leftarrow$ Sample R_g individuals (the new population) from T .

$D_g^N \leftarrow$ Mutate R_g individuals from D_g^N (optional).

EndRepeat

End.

Figure 4. Pseudocode for the BBIC algorithm.

```
R denotes an individual (chromosome).  
B denotes common and uncommon subsequences (Building Blocks).  
Algorithm Building_Block_Identification  
{The comparison of individual  $R_i$  to  $R_1, \dots, R_{i-1}$ }  
  For  $i = 2$  to (population_size / 2) do  
    For  $j = 1$  to ( $i-1$ ) do  
       $B \leftarrow$  Compare  $R_i$  to  $R_j$   
      (search for common and uncommon substructures)  
    EndFor  
  EndFor  
End.
```

Figure 5. Pseudocode for Building Block identification.

R denotes an individual (chromosome).

B_i denotes a set of common and uncommon subsequences (Building Blocks) that begin with the bit at position i .

C denotes a Fragment

Algorithm Building_Block_Composition

{The composition of the new individual R }

$i = 1$ {first position}

While $i < \text{individual_size}$ **do**

if $B_i \neq \emptyset$ **then** {if has one or more BBs }

$C \leftarrow \text{random}(B_i)$

$i \leftarrow i + \text{size}(C)$ {assign next position}

else {if there is no member}

$C \leftarrow \text{random}(0,1)$ {random from 0 or 1}

$i \leftarrow i + 1$

endif

$R \leftarrow R + C$ {concatenation}

EndWhile

$R \leftarrow \text{mutation}(R)$ {optional}

End.

Figure 6. Pseudocode for Building Block composition.

Desired BB	1	1	1	1	1	1	1	1
Pure BB	1	1	1	1	1	1	1	1
Mixed BB	1	1	1	0	1	1	1	1
Mixed BB	0	0	0	0	0	0	0	1
Non BB	0	0	0	0	0	0	0	0

Figure 8. Examples of BB classification.

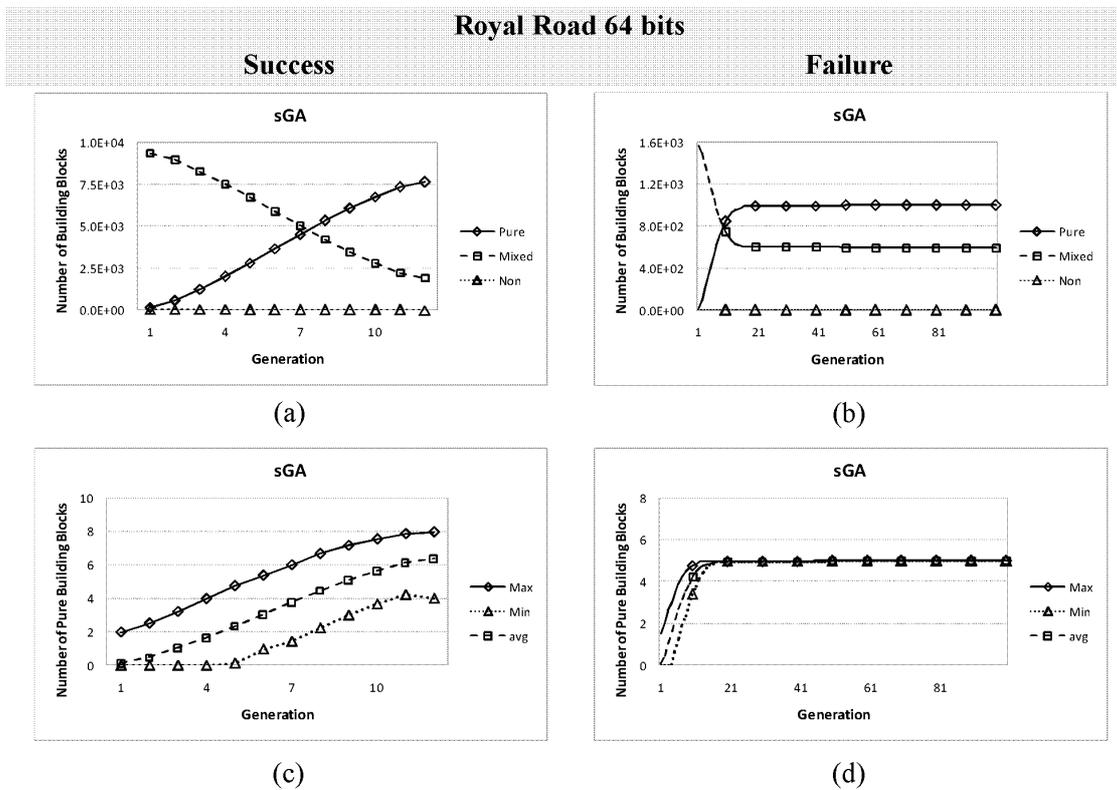


Figure 9. The sGA results for the Royal Road 64-bit problem: (a)(b) the number of BBs in each generation and (c)(d) the number of ideal BBs (Pure BBs) in each generation.

Royal Road 64 bits

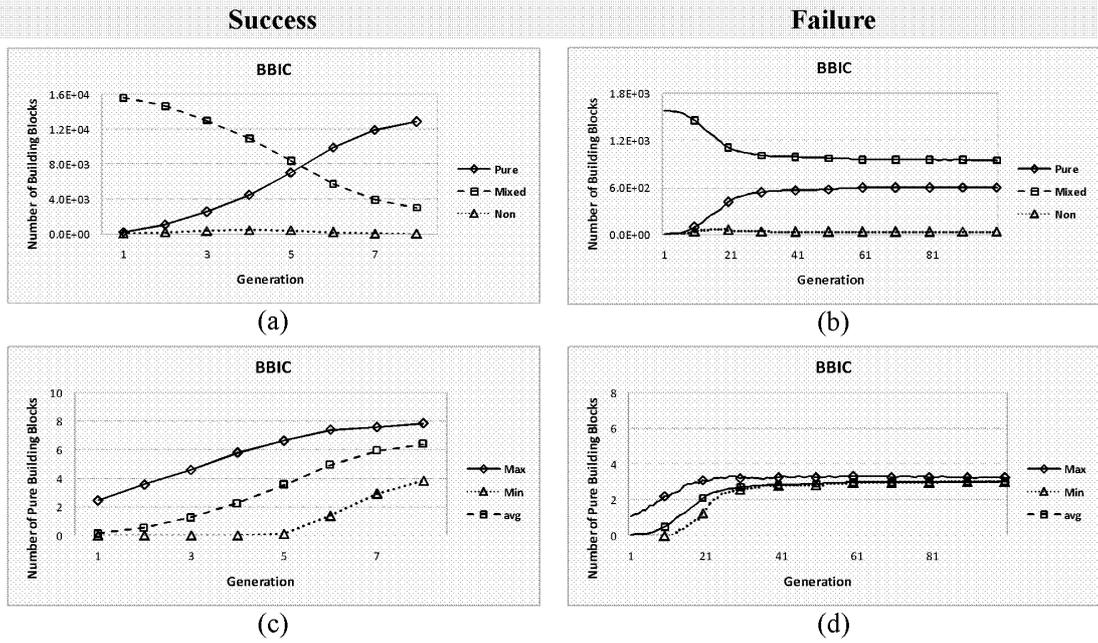
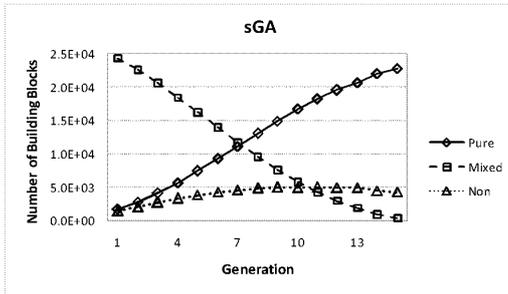


Figure 10. The BBIC algorithm results for the Royal Road 64-bit problem: (a)(b) the number of BBs in each generation, (c)(d) the number of ideal BBs (Pure BBs) in each generation.

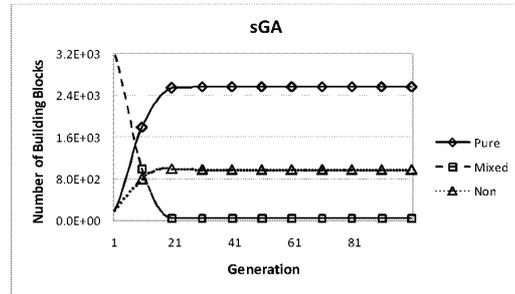
Trap-5 60 bits

Success

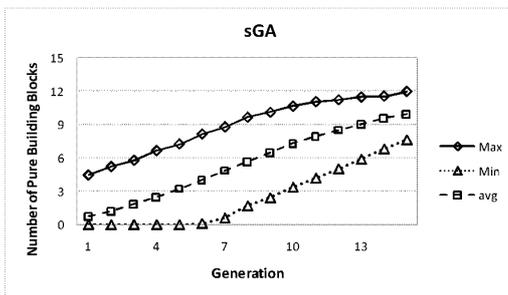
Failure



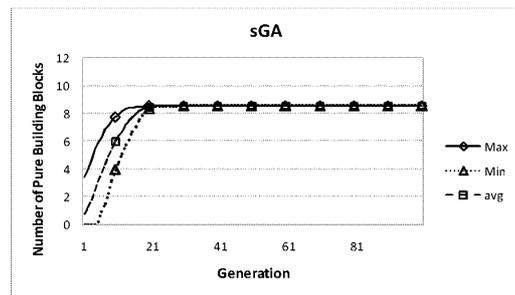
(a)



(b)



(c)



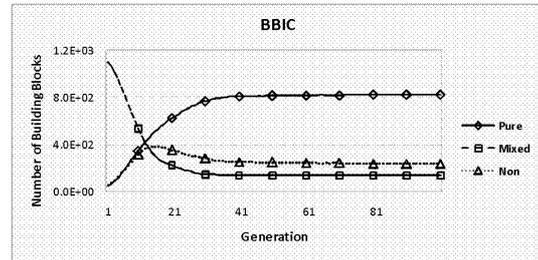
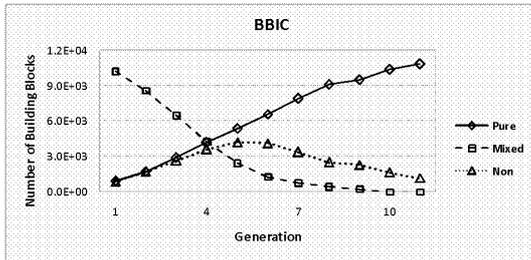
(d)

Figure 11. The sGA results for the Trap-5 60-bit problem: (a)(b) the number of BBs in each generation and (c)(d) the number of ideal BBs (Pure BBs) in each generation.

Trap-5 60 bits

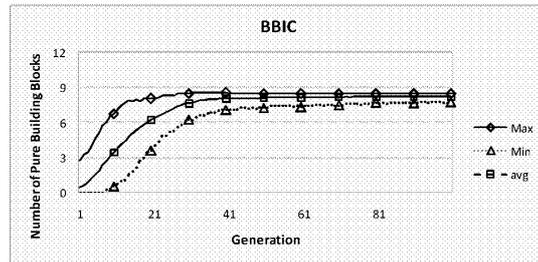
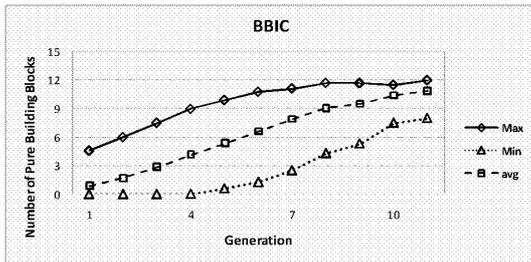
Success

Failure



(a)

(b)



(c)

(d)

Figure 12. The BBIC algorithm results for the Trap-5 60-bit problem: (a)(b) the number of BBs in each generation, (c)(d) the number of ideal BBs (Pure BBs) in each generation.

Royal Road 64 bits

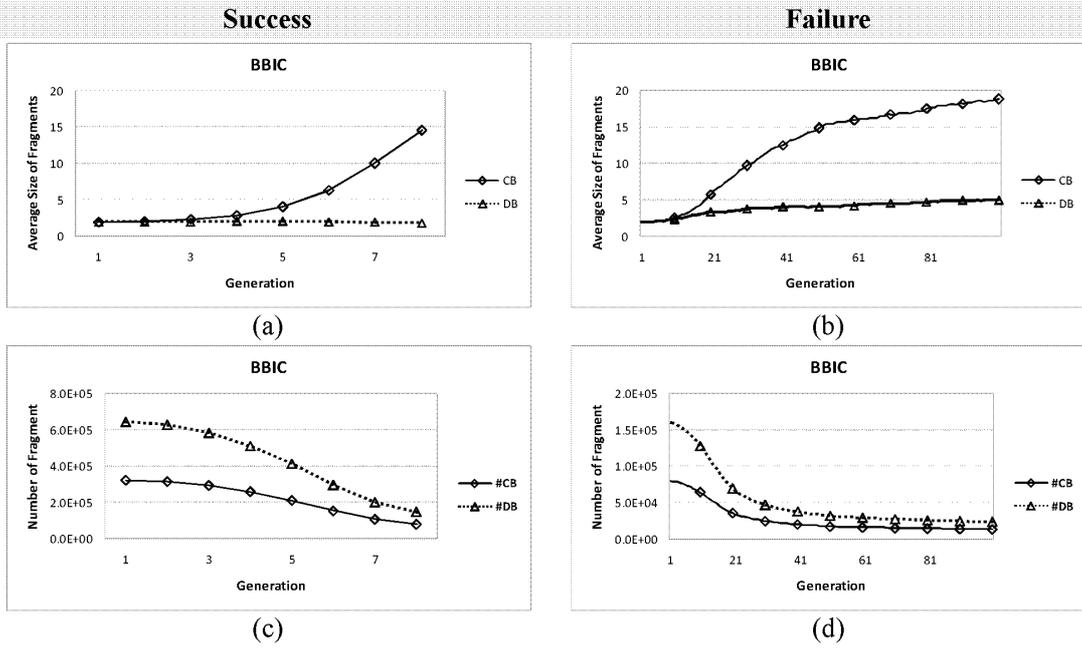


Figure 13 The BBIC algorithm results for the Royal Road 64-bit problem: (a)(b) the average size of the BBs in each generation and (c)(d) the number of BBs in each generation. Note: CB denotes the average common Building Block size, DB denotes the average different Building Block size, #CB denotes the number of common BBs and #DB denotes the number of different BBs.

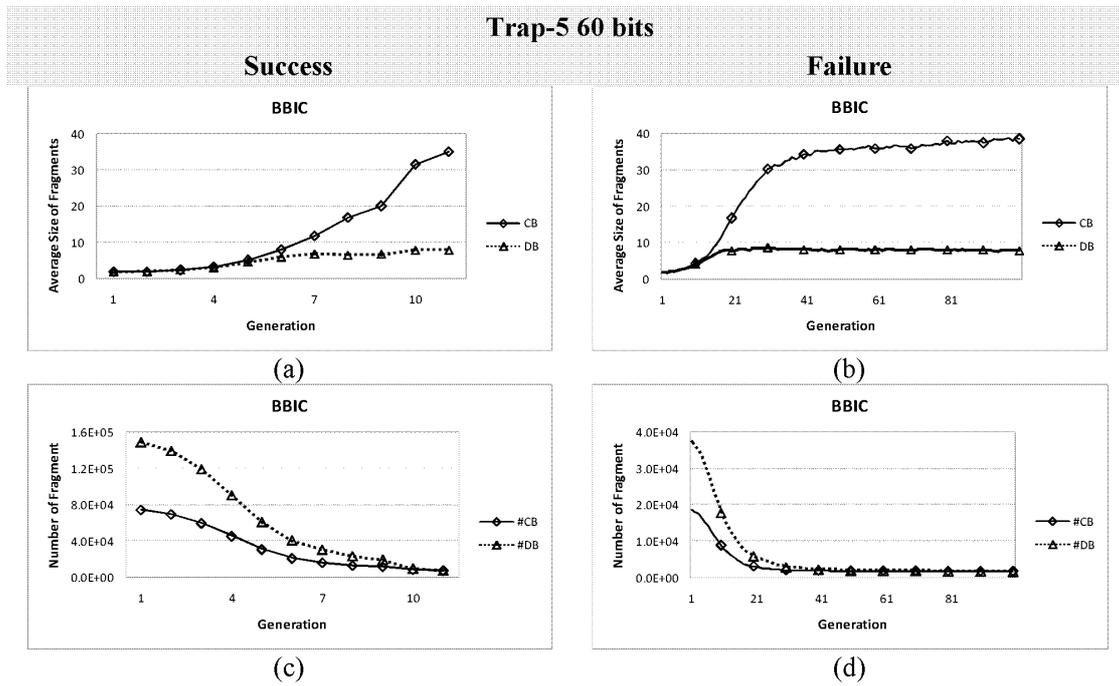


Figure 14 The BBIC algorithm results for the Trap-5 60-bit problem: (a)(b) the average size of the BBs in each generation and (c)(d) the number of BBs in each generation. Note: CB denotes the average common Building Block size, DB denotes the average different Building Block size, #CB denotes the number of common BBs and #DB denotes the number of different BBs.

Test Problems	Non-Deceptive		All Optimal bits 0/1	Straightforward	Hierarchical	Linear Fitness	Non-linear Fitness	Unimodal	Multimodal	BBs Test	Tightly Grouped BBs	Hierarchical BBs
	Non-Deceptive	Deceptive										
OneMax	●		●	●		●		●				
Royal Road	●		●	●		●		●		●	●	
Trap-5		●	●	●		●		●		●	●	
HIFF	●		●		●		●		●	●		●
hTrap-1		●	●		●		●	●		●	●	●

Figure 15. Characteristics of the benchmark problems.

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