Assignment I: Basic Quantum Computing

- Required:
  - ➢ Go to https://quantum-computing.ibm.com/
  - ➢ Register IBMid account or sign in with Google, Github, LinkedIn, or Twitter.
  - ➢ Download source codes at t.ly/FNXw and upload files "**Lab-1.ipynb**", "**Lab-2.ipynb**" and "**Lab-3.ipynb**" into IBM Quantum Lab.
- Assignments:
  - ➢ Lab-1: Operations on single qubit and multiple qubits gates by IBM Quantum.
  - ➢ Lab-2: Quantum circuits by IBM Quantum.
  - ➢ Lab-3: Superdense coding.

Quantum algorithms

- Deutsch-Jozsa algorithm

  ➤ We are given a hidden Boolean function f, which takes as input a string of bits, and returns either 0 or 1, that is:

 $$f(\{x_0, x_1, x_2, \dots\}) \to 0 \text{ or } 1 \text{ , where } x_n \text{ is } 0 \text{ or } 1$$

  ➤ The property of the given Boolean function is that it is guaranteed to either be balanced (returns 1 for half of the input domain and 0 for the other half) or constant (0 on all inputs or 1 on all inputs).

  ➤ Our task is to determine whether the given function is balanced or constant.

Quantum algorithms

- Deutsch-Jozsa algorithm

  ➢ For classical solution, we need to ask the oracle at least twice, but if we get twice the same output, we need to ask again. At most to query is $(N/2)+1$, where N is number of state.

  ➢ For quantum solution, need only one query. If the output is **the zero bit string**, we know that the oracle is **constant**. If it is **any other bit string**, we know that it is **balanced**.

  ➢ We have the function f implemented as a quantum oracle, which maps the state $|x\rangle|y\rangle$ to $|x\rangle|y \oplus f(x)\rangle$, where $\oplus$ is addition modulo 2.

Quantum algorithms

- Deutsch-Jozsa algorithm
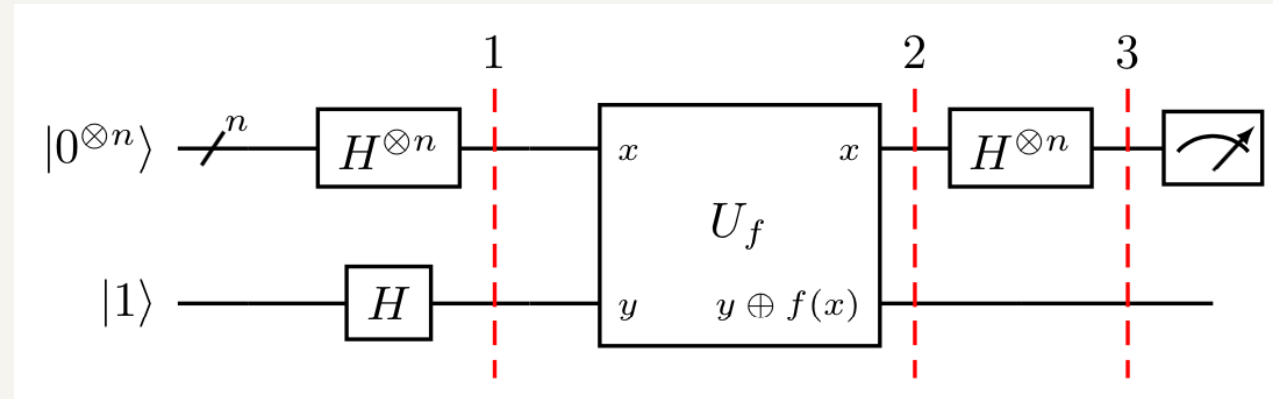
➢ The initial state of which can be expressed:

$$|\psi_0\rangle = |0\rangle^{\otimes n} |1\rangle$$

➢ which is then put into superposition, which can conveniently be expressed:

$$|\psi_1\rangle = \sum_{x \in \{0,1\}^n} \frac{1}{\sqrt{2^{n+1}}} |x\rangle \left(|0\rangle - |1\rangle\right)$$

Quantum algorithms

- Deutsch-Jozsa algorithm

  ➢ Apply the quantum oracle $|x\rangle|y\rangle$ to $|x\rangle|y\oplus f(x)\rangle$:

  $$|\psi_2\rangle = \sum_{x\in\{0,1\}^n} \frac{1}{\sqrt{2^{n+1}}}(-1)^{f(x)}|x\rangle(|0\rangle - |1\rangle)$$

  ➢ We now address the interference H^n on the first n wires, for which we use the expression:

  $$H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}}\sum_{z\in\{0,1\}^n}(-1)^{x\cdot z}|z\rangle$$

  ➢ which allows us to express:

  $$|\psi_3\rangle = \sum_{x\in\{0,1\}^n}\sum_{z\in\{0,1\}^n} \frac{1}{2^n}(-1)^{x\cdot z+f(x)}|z\rangle\frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

  ➢ where $x.z = x_0z_0\oplus x_1z_1\oplus\ldots\oplus x_{n-1}z_{n-1}$ is the sum of the bitwise product.

Quantum algorithms

- Deutsch-Jozsa algorithm

  ➤ We can now determine whether the function is constant or balanced by measuring the first *n* qubits of the final state.

  $$|\psi_3\rangle = \left(\sum_{z\in\{0,1\}^n} \sum_{x\in\{0,1\}^n} \frac{1}{2^n\sqrt{2}} (-1)^{x\cdot z + f(x)} |z\rangle \right) \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

  ➤ Specifically, we consider the probability of measuring zero on every qubit, which corresponds to the term in the superposition where $|x\rangle$ is $|0\rangle^{\otimes n}$

    - In the case where the function is constant, then the co-efficient of $|0\rangle^{\otimes n}$, $\sum_x (-1)^{f(x)}/2^n$ is equal to ± 1... as this has amplitude 1, then we measure $|0\rangle^{\otimes n}$ with probability one.

    - In the case where the function is balanced then $\sum_x (-1)^{f(x)}/2^n = 0$, and so we will never measure $|0\rangle^{\otimes n}$.

  ➤ So it follows that measuring the first *n* qubits allows us to determine with certainty whether the function is **constant (measure all zeros)** or **balanced (measure at least one 1)**.

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Quantum algorithms

- Deutsch-Jozsa algorithm
  - ➢ We can encode any mathematical function as a unitary matrix.
  - ➢ Deutsch's algorithm was the first algorithm that demonstrated a quantum advantage: specifically, a reduction in query complexity compared to the classical case.
  - ➢ The Deutsch-Jozsa algorithm generalises Deutsch's algorithm and reveals the possibility of exponential speed-ups using quantum computers.

Quantum algorithms

- Grover's algorithm

  ➢ It can be used to solve unstructured search problems in roughly $\sqrt{N}$ steps, where *N* is the amount of data.

  ➢ This algorithm can speed up an unstructured search problem quadratically using the amplitude amplification trick.

Quantum algorithms

- Grover's algorithm

  ➤ The example of Grover's algorithm for 3 qubits with two marked states $|101\rangle$ and $|110\rangle$.
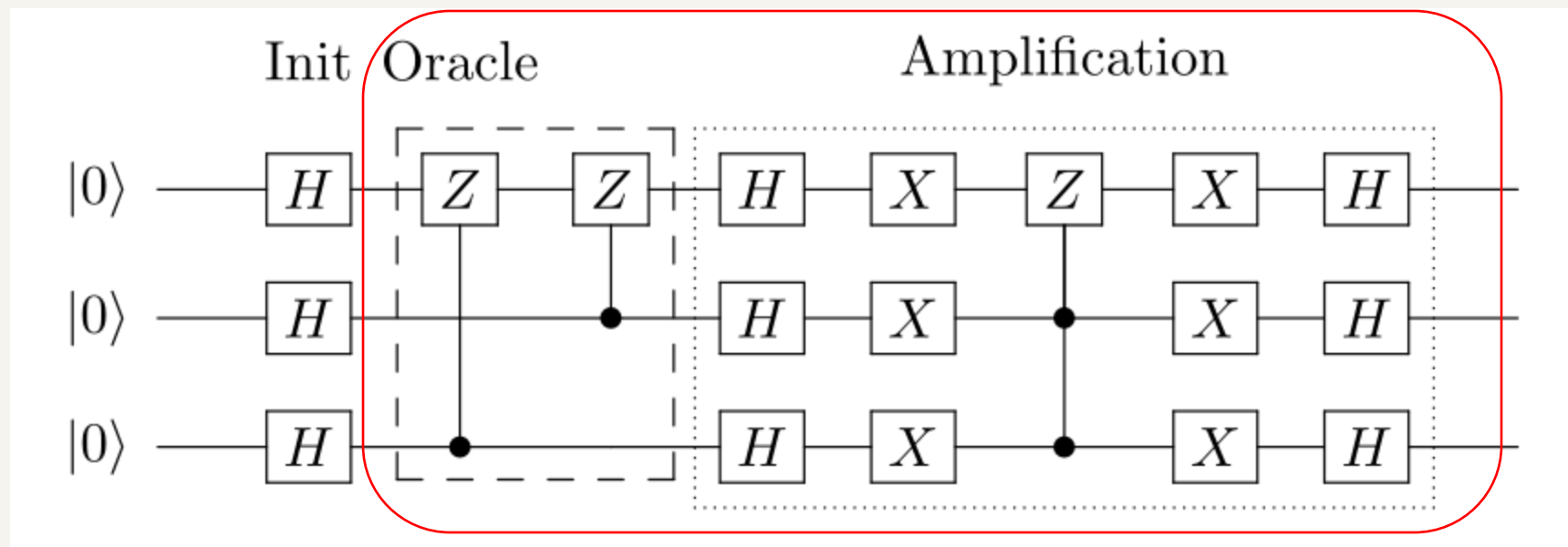
  Grover iterations $\sim \frac{\pi}{4}\sqrt{\frac{N}{t}}$

Photo courtesy of https://qiskit.org/textbook/ch-algorithms/grover.html

Quantum algorithms

- The implemented stages of the Grover's search algorithm:

  ➢ Initialization: In the first stage of the algorithm all qubits are set to be in superposition by applying the Hadamard gate to each qubit. After this operation the amplitude of each state is 1/sqrt(n).

  ➢ Oracle: The oracle function performs a phase flip on the marked state. If the marked state is $|0110\rangle$, the phase flip inverts the amplitude $\alpha 0110$ of the state.

  ➢ Amplification: The amplification stage performs an inversion of the average of the amplitudes.

  ➢ Measurement: The qubits are measured in finally.

Grover iterations ~ $\frac{\pi}{4}\sqrt{\frac{N}{t}}$

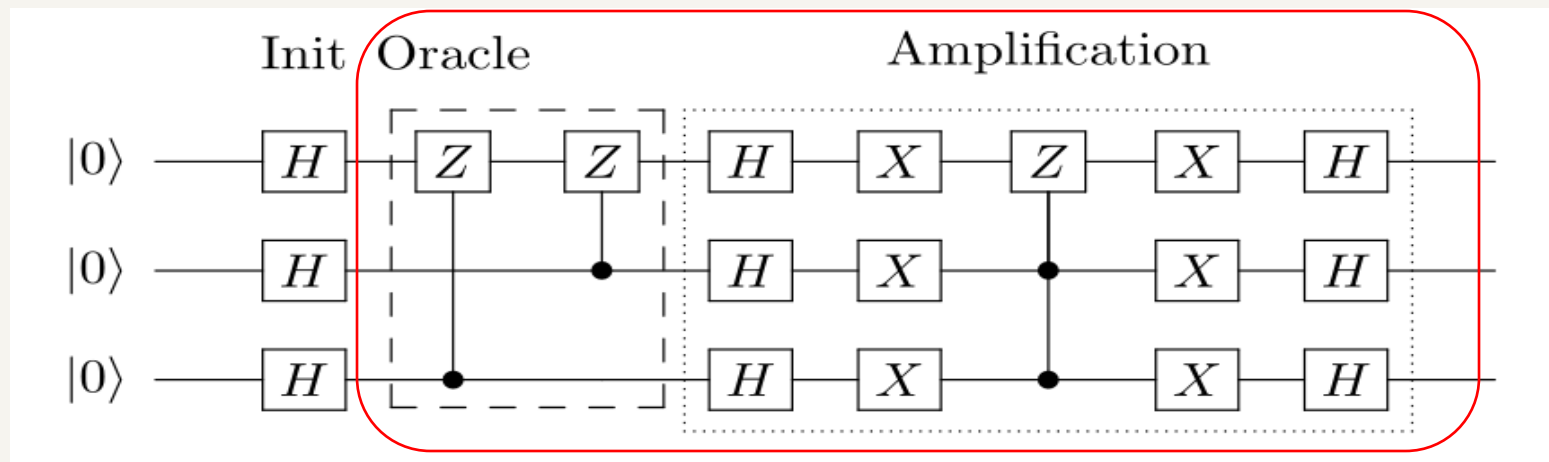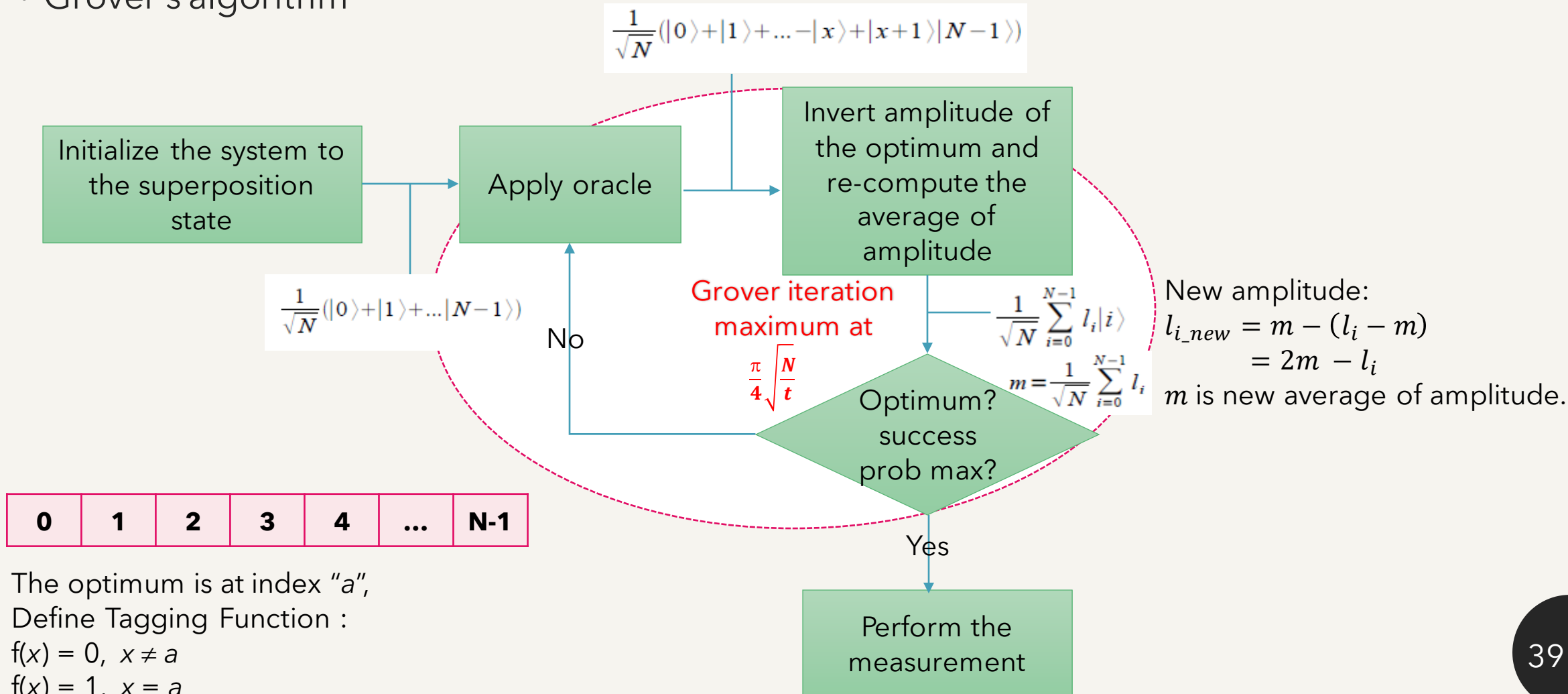Photo courtesy of https://qiskit.org/textbook/ch-algorithms/grover.html

Quantum algorithms

- Grover's algorithm

$$\frac{1}{\sqrt{N}}(|0\rangle + |1\rangle + \ldots - |x\rangle + |x+1\rangle |N-1\rangle)$$

| Initialize the system to the superposition state | → | Apply oracle | → | Invert amplitude of the optimum and re-compute the average of amplitude |

$$\frac{1}{\sqrt{N}}(|0\rangle + |1\rangle + \ldots |N-1\rangle)$$

Grover iteration maximum at

$$\frac{\pi}{4}\sqrt{\frac{N}{t}}$$

No

$$\frac{1}{\sqrt{N}}\sum_{i=0}^{N-1} l_i |i\rangle$$

$$m = \frac{1}{\sqrt{N}}\sum_{i=0}^{N-1} l_i$$

New amplitude:
$$l_{i\_new} = m - (l_i - m)$$
$$= 2m - l_i$$
m is new average of amplitude.

Optimum? success prob max?

Yes

Perform the measurement

| 0 | 1 | 2 | 3 | 4 | ... | N-1 |

The optimum is at index "*a*",
Define Tagging Function :
f(*x*) = 0, *x* ≠ *a*
f(*x*) = 1, *x* = *a*

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Quantum algorithms

- Operation of searching data by Grover's algorithm for 2 qubits:
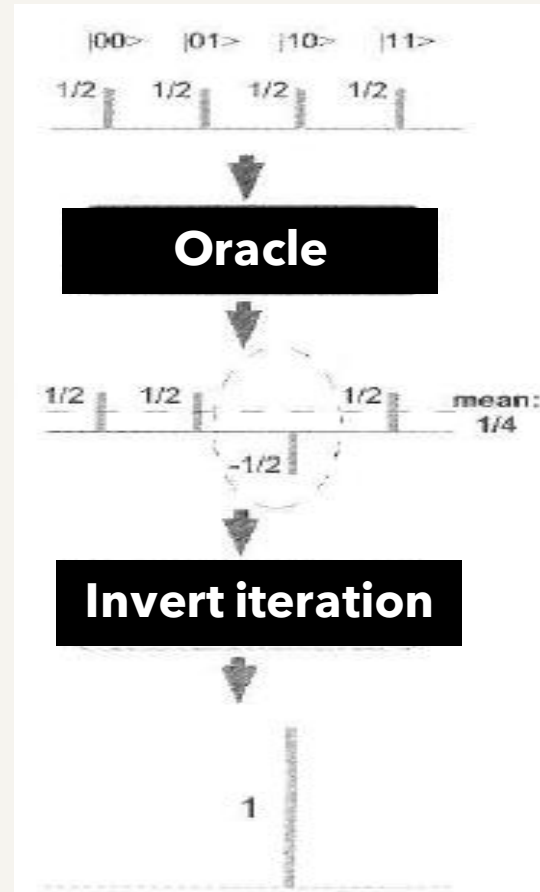
$$\frac{1}{2}(|00\rangle+|01\rangle+|10\rangle+|11\rangle)$$
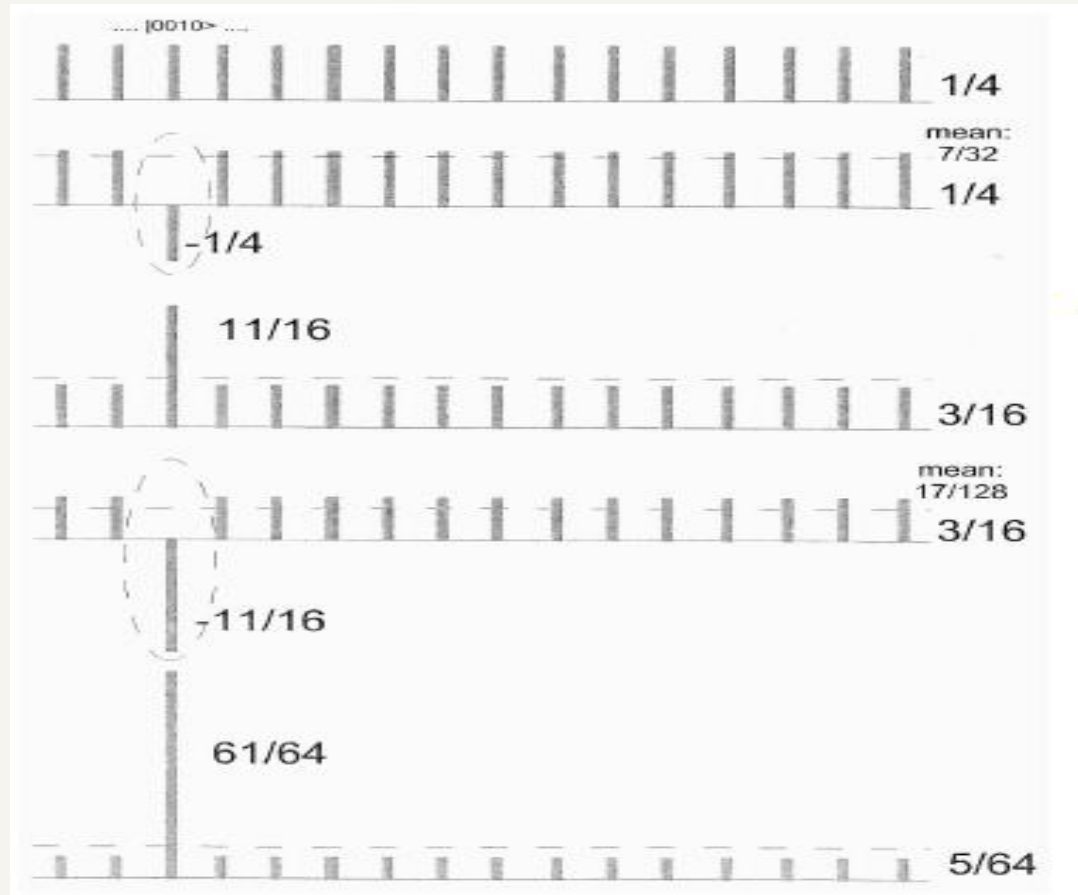
$$\frac{1}{2}(|00\rangle+|01\rangle-|10\rangle+|11\rangle)$$

$$m = \frac{\left(\frac{1}{2}+\frac{1}{2}-\frac{1}{2}+\frac{1}{2}\right)}{4} = \frac{1}{4}$$

$$l_i|00\rangle, |01\rangle, |11\rangle = \frac{1}{4}-\left(\frac{1}{2}-\frac{1}{4}\right)=0$$

$$l_i|10\rangle = \frac{1}{4}-\left(-\frac{1}{2}-\frac{1}{4}\right)=1$$

Quantum algorithms

- Operation of searching data by Grover's algorithm for 4 qubits:

Grover iteration = $\frac{\pi}{4} \; x \; \sqrt{\frac{N}{t}}$ times,

N is the number of data (states) and t is the number of target solutions.

Try it out at t.ly/FNXw *and upload files* "**Grover's algorithm.ipynb**" *into IBM Quantum Lab.*

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Assignment II: quantum algorithms

- Required:
  - ➢ Go to https://quantum-computing.ibm.com/
  - ➢ Download source codes at t.ly/FNXw and upload files "**Lab-4.ipynb**" into IBM Quantum Lab.
- Assignment:
  - ➢ Lab-4: Oracles and the Deutsch-Jozsa algorithm by IBM Quantum.