SIMULATED ANNEALING

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Simulated Annealing

- a combinatorial optimization method
- effectively solved the famous traveling salesman problem
- the implementation is quite simple
- an analogy to the statistical mechanics of annealing in solids

Combinatorial Optimization Problems

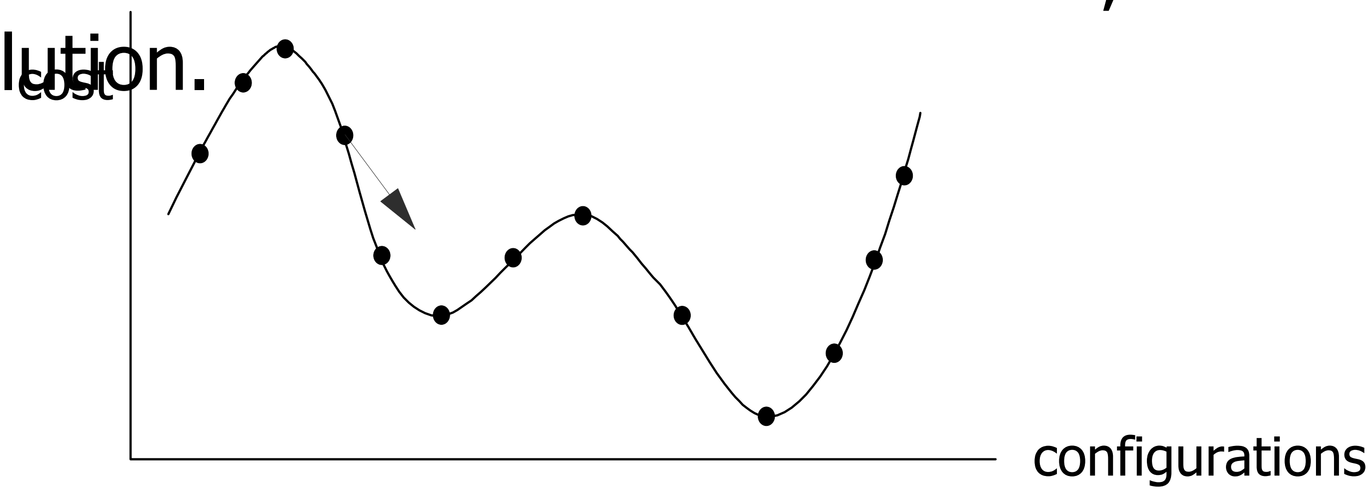
- An objective function to be minimized.
- The solution space is not the N-dimensional space of N *continously variable* parameters.
- The solution space is *discrete*, and very large (exponentially or factorially large).
- The space can not be exhaustively explored.

The Traveling Salesman Problem

- Given *N* cities with given position (x_i , y_i).
- The salesperson visits all cities and returns to his or her city of origin.
- Each city is to be visited once.
- The route is to be made as short as possible
- The problem belongs to a class known as an *NP-Complete* problems.
  (Obtaining an *exact* solution requires an exponentially increasing number of steps as *N* becomes larger.)

Heuristic Strategies

- Constructive heuristics
  - build up a good answer directly, piece by piece.
- Iterative improvement
  - attempt to *perturb* existing suboptimal solution in the direction of a better, lower-cost solution.



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Simulated Annealing

```
T = starting temperature
do {
  do {
    Generate a random move;
    Evaluate the change in energy ΔE;
    if ( ΔE < 0 ) {
      accept this move, and update config.
    } else {
      accept the move with probability
      P = exp(-ΔE/kT)
      update configuration if accepted;
    }
  } until ( #loops > 100N or #changes > 10N )
  T = 0.9T;
} until ( reduce E is discouraging );
```

downhill move

uphill move

annealing schedule

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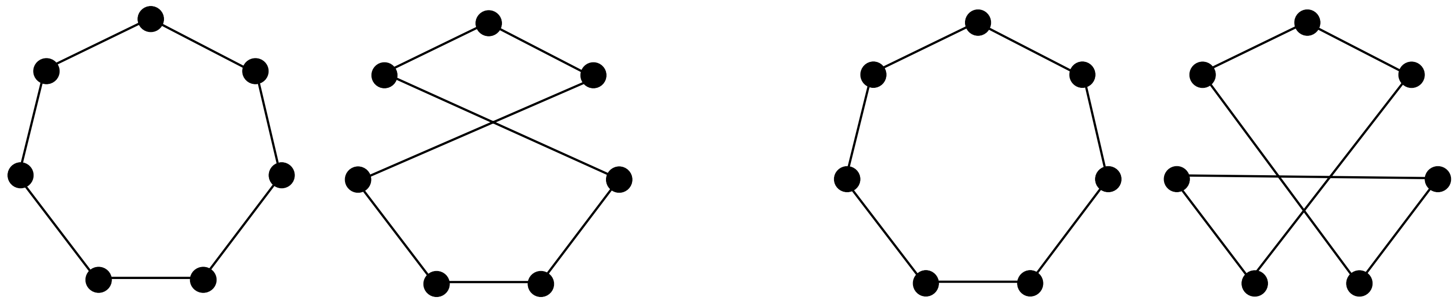
Simulated Annealing in TSP

- **Configuration** : a permutation of the number $1, ..., N$, the order in which the cities are visited.
- **Rearrangement** :



- **Objective function** : $E = \sum_{i=1}^{N} \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$
- **Annealing schedule** :
  – Starting with a large T value, then decreasing T downward to 90% of the previous T
  – hold each T constant for $100N$ reconfigurations or for $10N$ successful reconfigurations

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Cost vs. Temperature