Proof by contradiction examples

Example: Proof that $\sqrt{2}$ is irrational.

Solution:

Assume the negation, that is $\sqrt{2}$ is rational.

From this assumption, $\sqrt{2}$ can be written in terms of $\frac{a}{b}$, where a and b have no common factor. This means $\frac{a}{b}$ is in lowest terms.

$$\begin{aligned} \sqrt{2} &= \frac{a}{b} \\ 2 &= \frac{a^2}{b^2} \\ 2b^2 &= a^2 \end{aligned}$$

This means a^2 is even, which implies that a is even since "odd*odd = odd".

And since we get that a is even, a can be written as $2c$.

Back to the equation again, now we get

$$2b^2 = (2c)^2$$

$$\begin{aligned} 2b^2 &= 4c^2 \\ b^2 &= 2c^2 \end{aligned}$$

This means, similar to the case of a, that b is even.

But now a and b have a common factor because they are both even. This is contradiction.

Our assumption must be false. Thus the thing we want to prove is true because it has an opposite truth value to the assumption.

Example: Proof that at least 4 of any 22 days must fall on the same day of the week.

Solution:

Assume: at most 3 of any 22 days must fall on the same day of the week.

One week = 7 days...so in fact there can be 4 on the same day.. (draw picture).

Therefore the assumption is false and its negation (which is what we want to prove) is true.

Example: Proof the following: If 3n+2 is odd then n is odd

Solution: The statement is (3n+2 is even) $\vee$ (n is odd).

Assume: (3n+2 is odd) $\wedge$ (n is even).

But it can be seen that if n is even, 3n+2 must be even.... Therefore, contradiction.